On the separation of hydrodynamic and acoustic waves in linear free-shear flows

A. Agarwal, G. Gabard, S. Sinayoko and Z. Hu

University of Southampton
Institute of Sound and Vibration Research

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Outline

1. Introduction
   - Objective
   - Motivation
   - Introduction to filtering in time domain
   - Filter characteristics

2. Wave-operator filter
   - The wave-operator filter
   - Filtering of a two-dimensional shear layer problem

3. Corrective filter
   - Rationale
   - Proof of concept based on the two-dimensional shear layer problem
   - General solution based on Green’s functions
Introduction

Objective

Motivation

Introduction to filtering in time domain

Filter characteristics

Wave-operator filter

The wave-operator filter

Filtering of a two-dimensional shear layer problem

Corrective filter

Rationale

Proof of concept based on the two-dimensional shear layer problem

General solution based on Green’s functions
Filter out the acoustic waves
Leave the hydrodynamic waves unchanged
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Motivation

Navier-Stokes equations

\[ \mathbf{Nv} = \mathbf{s} \quad (1) \]

Filtered Navier-Stokes equations

\[ \mathbf{N\tilde{v}} = \mathbf{\tilde{s}} \quad (2) \]

Linearisation given by Eq. (1) - Eq. (2)

\[ \mathbf{Lv'} = \mathbf{s} - \mathbf{\tilde{s}} \approx f(\mathbf{\tilde{s}}) \quad (3) \]

\[ f(\mathbf{\tilde{s}}) \equiv \text{“true sources of sound”} \quad \text{[Goldstein, 2005]} \]
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Introduction to filtering in time domain

Flow decomposition

\[ p = \tilde{p} + p' \]

- \( \tilde{p} \): base flow obtained by filtering \( p \)
- \( p' \): fluctuating part

What we want

- \( \tilde{p} \): no acoustic fluctuations \(\Rightarrow\) non-propagating base flow
- \( p' \): acoustic fluctuations only.
Flow decomposition

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Convolution filter example: 1 dimensional signal

Moving average filter: $\tilde{s} = h_{MA} \ast s$
Introduction to filtering in time domain

Convolution filter example: 2 dimensional signal

\[ s \]

Gaussian filter \( \tilde{s} = h \ast s \)
Introduction to filtering in time domain

Differential filter example

\[ \tilde{s} = \nabla^2 s \]
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Filter characteristics

Defining property

\[ \tilde{P}(k, \omega) = 0 \quad \text{for} \quad |k| = \frac{\omega}{c_0} \]

Other requirements

- Causality
- Easy to implement
Filter characteristics

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The wave-operator filter

Time domain

\[ \tilde{p}(\mathbf{x}, t) = \left( \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p(\mathbf{x}, t), \]

Frequency domain

\[ \tilde{P}(\mathbf{k}, \omega) = \left( |\mathbf{k}|^2 - \frac{\omega^2}{c_0^2} \right) P(\mathbf{k}, \omega) \]

\[ \Rightarrow \tilde{P}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_0} \]
The wave-operator filter

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Filtering of a two-dimensional problem
Parallel flow & source definitions

- $M_j = 0.756$
- $T_j = 600$ K
- Gaussian harmonic energy source, $\omega_0 = 76$ rad/s
Filtering of a two-dimensional problem

Results

- acoustic waves are filtered successfully
- hydrodynamic waves are distorted
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Rationale

Inverse filtering in frequency domain

1. \[ \tilde{P}(k, \omega) = \left( |k|^2 - \frac{\omega^2}{c_0^2} \right) P(k, \omega) \]

2. \[ \hat{P}(k, \omega) = \frac{1}{\left( |k|^2 - \frac{\omega^2}{c_0^2} \right)} \tilde{P}(k, \omega) \]

Convolution filtering

1. Time domain: \[ \hat{\rho} = h \ast \tilde{\rho} \]

2. Frequency domain: \[ \hat{P} = H \tilde{P} \]

\[ \Rightarrow H = \frac{1}{\left( |k|^2 - \frac{\omega^2}{c_0^2} \right)} \]
Rationale

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Proof of concepts based on the two-dimensional shear layer problem

Two dimensional shear layer problem

- $k_x = \text{constant} = k_{x_0}$
- $\omega = \text{constant} = \omega_0$

\[ h(x, t) = \delta(x)\delta(t)\frac{e^{-\kappa|y|}}{2\kappa} \]
Proof of concepts based on the two-dimensional shear layer problem

Corrective filter

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Proof of concepts based on the two-dimensional shear layer problem

Results

\[ \Rightarrow \text{Reconstruction of the hydrodynamic wave from the filtered pressure seems possible.} \]
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General solution based on Green’s function

Green’s function

Wave-operator filtering

\[ \Box^2 p = \tilde{p} \]

- \( \Box^2 \) denotes the wave operator
- \( \tilde{p} \) is the source term

Inverse filtering with Green’s function

\[ p = G \ast \tilde{p}, \]

- \( G \) is a free field Green’s function for operator \( \Box^2 \).
General solution based on Green’s function

Corrective filter in two and three dimensions

Corrective filter in two dimensions

\[ \hat{p}(\mathbf{x}, t) = \int_{S} \int_{0}^{+\infty} \frac{\tilde{p}(\mathbf{x} - \mathbf{x}', t - t')} {2\pi \sqrt{t'^2 - |\mathbf{x}'|^2 / c_0^2}} \, dt' \, d^2\mathbf{x}' , \]

Corrective filter in three dimensions

\[ \hat{p}(\mathbf{x}, t) = \int_{V} \frac{\tilde{p}(\mathbf{x} - \mathbf{x}', t - |\mathbf{x}'|/c_0)} {4\pi |\mathbf{x}'|} \, d^3\mathbf{x}' . \]
Summary

- Wave-operator allows to filter acoustic fluctuations easily
- It distorts the hydrodynamic fluctuations
- A corrective filter based on Green’s function could be used to restore the hydrodynamic fluctuations.
For further reading


Aerodynamic sound; Space-time filtering; Non-radiating components;