On the separation of hydrodynamic and acoustic waves in linear free-shear flows

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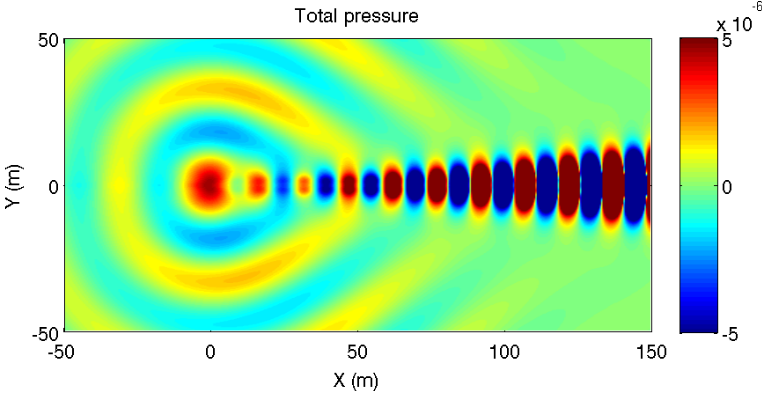
Outline

1. Introduction
   - Objective
   - Motivation
   - Introduction to filtering in time domain
   - Filter characteristics
2. Wave-operator filter
   - The wave-operator filter
   - Filtering of a two-dimensional shear layer problem
3. Corrective filter
   - Rationale
   - Proof of concept based on the two-dimensional shear layer problem
   - General solution based on Green's functions

Outline

Objective



- filter out the acoustic waves
- leave the hydrodynamic waves unchanged

Outline

Motivation

Navier-Stokes equations

$$\mathbf{N}\mathbf{v} = \mathbf{s} \tag{1}$$

Filtered Navier-Stokes equations

$$\mathbf{N}\tilde{\mathbf{v}} = \tilde{\mathbf{s}} \tag{2}$$

Linearisation given by Eq. (1) - Eq. (2)

$$\mathbf{L}\mathbf{v}' = \mathbf{s} - \tilde{\mathbf{s}} \approx f(\tilde{\mathbf{s}}) \tag{3}$$

$f(\tilde{\mathbf{s}}) \equiv$ "true sources of sound" [Goldstein, 2005]

Outline

Introduction to filtering in time domain

Flow decomposition

Flow decomposition

$$p = \tilde{p} + p'$$

$\tilde{p}$ base flow obtained by filtering p

p' fluctuating part

What we want

- $\tilde{p}$: no acoustic fluctuations $\Rightarrow$ non-propagating base flow
- p': acoustic fluctuations only.

Introduction to filtering in time domain

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Introduction to filtering in time domain

Convolution filter example

Convolution filter example: 1 dimensional signal

Moving average filter: $\tilde{s} = h_{MA} * s$

Introduction to filtering in time domain

Convolution filter example

Convolution filter example: 2 dimensional signal

s

Gaussian filter $\tilde{s} = h * s$

Introduction to filtering in time domain

Differential filter example

Differential filter example

s

$\tilde{s} = \nabla^2 s$

Outline

Filter characteristics

Defining property

$$\tilde{P}(\mathbf{k},\omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_0}$$

Other requirements

- Causality
- Easy to implement

Filter characteristics

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$$\tilde{P}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_0}$$

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The wave-operator filter

Time domain

$$\tilde{p}(\mathbf{x}, t) = \left(\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p(\mathbf{x}, t),$$

Frequency domain

$$\tilde{P}(\mathbf{k}, \omega) = \left(|\mathbf{k}|^2 - \frac{\omega^2}{c_0^2} \right) P(\mathbf{k}, \omega)$$

$$\Rightarrow \quad \tilde{P}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_0}$$

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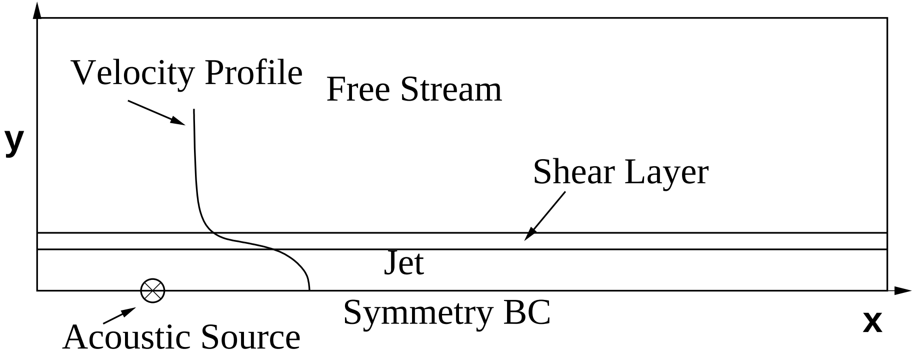
Filtering of a two-dimensional problem

Parallel flow & source definitions

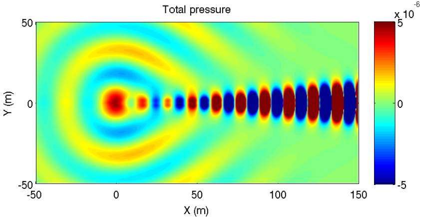


- $M_j = 0.756$
- $T_j = 600$ K
- Gaussian harmonic energy source, $\omega_0 = 76\mathrm{rad/s}$

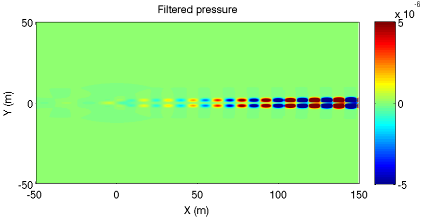
Filtering of a two-dimensional problem

Results

Pressure p



Filtered pressure $\tilde{p}$



Results

- acoustic waves are filtered successfully
- hydrodynamic waves are distorted

Outline

Rationale

Inverse filtering in frequency domain

1. $\tilde{P}(\mathbf{k},\omega) = \left(|\mathbf{k}|^2 - \frac{\omega^2}{c_0^2}\right) P(\mathbf{k},\omega)$
2. $\hat{P}(\mathbf{k},\omega) = \frac{1}{\left(|\mathbf{k}|^2 - \frac{\omega^2}{c_0^2}\right)} \tilde{P}(\mathbf{k},\omega)$

Convolution filtering

1. Time domain: $\hat{p} = h * \tilde{p}$
2. Frequency domain: $\hat{P} = H\tilde{P}$

$$\Rightarrow \quad H = \frac{1}{\left(|\mathbf{k}|^2 - \frac{\omega^2}{c_0^2}\right)}$$

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Proof of concepts based on the two-dimensional shear layer problem

Corrective filter

Two dimensional shear layer problem

- $k_x = \text{constant} = k_{x_0}$
- $\omega = \text{constant} = \omega_0$

$\Rightarrow \; h(\mathbf{x}, t) = \delta(x)\delta(t)\frac{e^{-\kappa|y|}}{2\kappa}$

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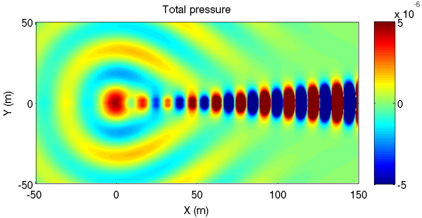
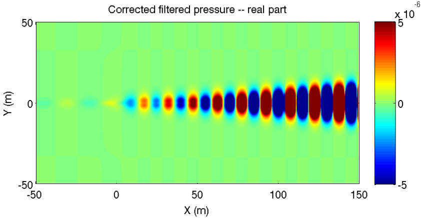


Corrected filtered pressure $\hat{p}$



$\Rightarrow$ Reconstruction of the hydrodynamic wave from the filtered pressure seems possible.

Outline

General solution based on Green's function

Green's function

Wave-operator filtering

$$\Box^2 p = \tilde{p}$$

- $\Box^2$ denotes the wave operator
- $\tilde{p}$ is the source term

Inverse filtering with Green's function

$$p = G * \tilde{p},$$

G is a free field Green's function for operator $\Box^2$.

General solution based on Green's function

Corrective filter in two and three dimensions

Corrective filter in two dimensions

$$\hat{p}(\mathbf{x},t) = \int_{\mathcal{S}} \int_{\frac{|\mathbf{x}'|}{c_0}}^{+\infty} \frac{\tilde{p}(\mathbf{x}-\mathbf{x}', t-t')}{2\pi\sqrt{t'^2 - |\mathbf{x}'|^2/c_0^2}}\, dt'\, d^2\mathbf{x}',$$

Corrective filter in three dimensions

$$\hat{p}(\mathbf{x},t) = \int_{V} \frac{\tilde{p}(\mathbf{x}-\mathbf{x}', t-|\mathbf{x}'|/c_0)}{4\pi|\mathbf{x}'|}\, d^3\mathbf{x}'.$$

Summary

- Wave-operator allows to filter acoustic fluctuations easily
- It distorts the hydrodynamic fluctuations
- A corrective filter based on Green's function could be used to restore the hydrodynamic fluctuations.

For further reading

Goldstein, M. (2003).
A generalized acoustic analogy.
Journal of Fluid Mechanics, 488:315 – 33.

Goldstein, M. (2005).
On identifying the true sources of aerodynamic sound.
Journal of Fluid Mechanics, 526:337 – 347.
Aerodynamic sound;Space-time filtering;Non-radiating components;.