Noise prediction for serrated trailing-edges

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June 22, 2015
Outline

Motivation
  Why is TE noise important?

Introduction to TE noise generation and control
  TE noise generation
  TE noise control
  Inaccurate existing model

Analytical formulation
  The mathematical model
  Fourier transformation and iterative-solving procedure
  Far-field sound

Results
  FEM validation
  Model results
  Comparison with Howe’s model
  Noise reduction mechanisms

Conclusion
TE noise problems are important

**Figure 1:** Applications where TE noise is important

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2. Fig(b): blog.journals.cambridge.org/2013/01/wind-turbine-syndrome-fact-or-fiction
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TE noise problems are important

- TE noise of an approaching aircraft

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- TE noise of rotating fans

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TE noise generation

Focus on the turbulent-boundary-layer TE noise, which will be referred to as TE noise.
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When the turbulent boundary layer convects past the TE, the non-radiating pressure fluctuation is scattered into sound capable of propagating to the far-field.

Figure 2: TE noise generation by edge-scattering
TE noise reduction techniques

Figure 3: TE noise reduction techniques

4 5 6 7

4 Fig(a): T.Geyer et al 2010
5 Fig(b): Michaela Herr et al 2005
6 Fig(c): Gruber’s PhD thesis 2012
7 Theory: (a) and (b) Jaworski and Peake 2013, Lorlna Ayton (c) Howe 1991
TE noise reduction techniques

- Porous airfoil

Figure 3: TE noise reduction techniques

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TE noise reduction techniques

- Porous airfoil
- Brush-type TE

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TE noise reduction techniques

- Porous airfoil
- Brush-type TE
- Serrated TEs

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Howe’s model

Howe’s model significantly overpredicts the noise reduction capability of serrated TEs.

Figure 4: Comparison of experiment and Howe’s model
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The mathematical model

Figure 5: The schematic of a flat plate with a serrated TE

The following wave equation needs to be solved (Roger and Moreau 2013)

\[
\left( \beta^2 + H'^2(y) \right) \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - 2H'(y) \frac{\partial^2 P}{\partial x \partial y} + \\
\left( 2iM_0k - H''(y) \right) \frac{\partial P}{\partial x} + k^2 P = 0,
\]

(1)
Fourier transformation

Making use of Fourier transformation

\[ P(x, y, z) = \sum_{-\infty}^{\infty} P_n(x, z)e^{ik_{2n}y}, \]  

(2)

where, \( k_{2n} = k_2 + 2n\pi/\lambda, \)
Fourier transformation

Making use of Fourier transformation

\[ P(x, y, z) = \sum_{-\infty}^{\infty} P_n(x, z) e^{ik_2ny}, \quad (2) \]

where, \( k_{2n} = k_2 + 2n\pi/\lambda \), the wave equation reduces to

\[ \mathcal{D}P = AP + B \frac{\partial P}{\partial x}, \quad (3) \]

where,

\[ \mathcal{D} = \left\{ (\beta^2 + \sigma^2) \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + 2ikM_0 \frac{\partial}{\partial x} \right\}. \quad (4) \]
The iterative-solving procedure

$P^{(0)}$ is obtained by solving

$$\mathcal{D}P = AP.$$  \hspace{1cm} (5)
The iterative-solving procedure

\( \mathbf{P}^{(0)} \) is obtained by solving

\[
\mathcal{D} \mathbf{P} = A \mathbf{P}.
\] (5)

Then \( \mathbf{P}^{(1)} \) is evaluated by solving

\[
\mathcal{D} \mathbf{P} = A \mathbf{P} + \mathbf{B} \frac{\partial \mathbf{P}^{(0)}}{\partial x}.
\] (6)
The iterative-solving procedure

\( P^{(0)} \) is obtained by solving

\[ \mathcal{D}P = AP. \]  \hspace{1cm} (5)

Then \( P^{(1)} \) is evaluated by solving

\[ \mathcal{D}P = AP + B \frac{\partial P^{(0)}}{\partial x}. \]  \hspace{1cm} (6)

Continuing this process, \( P^{(2)} \) is found by solving

\[ \mathcal{D}P = AP + B \frac{\partial P^{(1)}}{\partial x}. \]  \hspace{1cm} (7)
The iterative-solving procedure

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A solution sequence

\[ P^{(0)}, P^{(1)}, P^{(2)}, P^{(3)} \ldots \]
Far-field sound

The far-field sound is obtained by evaluating the surface integral based on the theories of Kirchoff and Curle.

\[ p_f(x, \omega) = \frac{-i \omega x_3}{4\pi c_0 S_0^2} \int \int_s \Delta P(x', y') e^{-ikR} \, dx' \, dy', \quad (8) \]

where \( S_0^2 = x_1^2 + \beta^2(x_2^2 + x_3^2) \), and \( R \) takes the following form:

\[ R = \frac{M_0(x_1 - x') - S_0}{\beta^2} + \frac{x_1 x' + x_2 y' \beta^2}{\beta^2 S_0}, \quad (9) \]

where, \( \Delta P \) denotes the pressure jump across the flat plate.
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FEM validation

For wide serrations

Figure 6: SPL at $90^\circ$ above the trailing-edge in the mid-span plane with $x_3 = 1$ due to a wall pressure gust of frequency $\omega$ with $k_2 = 0$, parameters of the serrations are $\lambda/h = 6, h/c = 0.025$. 
FEM validation cont.

For narrow serrations,

\[
20 \log_{10} |p_f| (\text{dB})
\]

\[
\begin{array}{cccc}
\text{Baseline-theory} & \text{Baseline-FEM} & \text{Serrated-theory} & \text{Serrated-FEM} \\
\end{array}
\]

(a) \(\lambda/h = 3\)

(b) \(\lambda/h = 1\)

Figure 7: SPL at 90° above the trailing-edge in the mid-span plane with \(x_3 = 1\) due to a wall pressure gust of frequency \(\omega\) with \(k_2 = 0\), parameters of the serrations are \(h/c = 0.05\) with \(M_0 = 0.1\).
Model results

For wide serrations,

(a) $\lambda/h = 8$

(b) $\lambda/h = 4$

Figure 8: The normalized spectrum for straight and serrated trailing-edges, $h/c = 0.025$, $M_0 = 0.1$, the observer is at $90^\circ$ above the trailing-edge in the mid-span plane with $x_3 = 1\lambda/h = 8$, $\lambda/h = 4$. 
Model results cont.

For narrow serrations,

\[
\begin{align*}
\lambda/h &= 0.4 \\
\lambda/h &= 0.2
\end{align*}
\]

Figure 9: The normalized spectrum for straight and serrated trailing-edges, \( h/c = 0.05, M_0 = 0.1 \), the observer is at \( 90^\circ \) above the trailing-edge in the mid-span plane with \( x_3 = 1 \).
Comparison with Howe’s model

Figure 10: The normalized spectrum of Howe’s model and the new model, $h/c = 0.05$, $M_0 = 0.1$, the observer is at $90^\circ$ above the trailing-edge in the mid-span plane with $x_3 = 1$. (a) $\lambda/h = 0.4$, $h/c = 0.05$, $M_0 = 0.1$ (b) $\lambda/h = 0.2$, $h/c = 0.05$, $M_0 = 0.1$
Noise reduction mechanism

\[ \sigma = 5 \]

(a) \( k\lambda = \pi/10 \), (b) \( k\lambda = \pi/5 \), (c) \( k\lambda = \pi/2 \), (d) \( k\lambda = \pi \)

Figure 9. The scattered surface pressure distribution at different frequencies for the same serration of \( 4h/\lambda = 5 \).

Figure 10. Scattered pressure on the serrated edge for different frequencies. (a) real part; (b) imaginary part.
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1. Compared to Howe’s model, the presented model includes the convection effect of the mean flow, and can better agrees with experiments.
2. It is found that the destructive interference of the scattered pressure is the cause of sound reduction.
3. The approach used in this model can be used for other serrations. Future work on optimizing the serration profiles can be done.
Thank You!