A combined FEM/Radiating-surface approach for noise propagation in unbounded domains with mean flow

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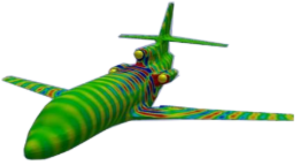
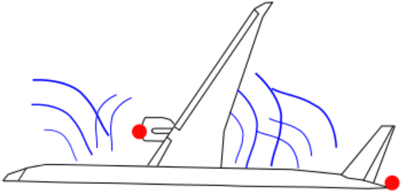
Aircraft large-scale noise propagation

Motivation:

- Prediction of noise radiation for design and certification
- Impact of non-uniform mean flow on noise propagation
- Computational cost of far-field noise prediction with mean flow





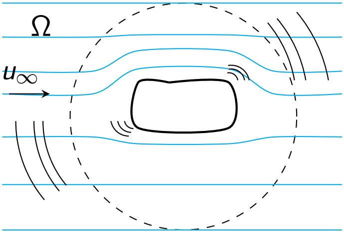
Outline

- Background
- Numerical method
- Error analysis
- Numerical results
- Concluding remarks

A hybrid FEM/Integral-formulation

Why a hybrid method for noise propagation with mean flow?

- FE solutions suffer from dispersion error and pollution effects
- Integral formulation inherently satisfies Sommerfeld condition
- Complex boundary and transmission conditions when Lorentz transformation is applied to the integral formulation with mean flow.



Numerical method

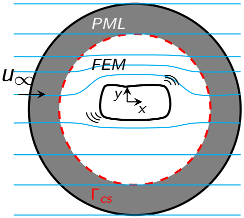
1) Near field: **FEM + PML** for scattering and refraction on a non-uniform flow

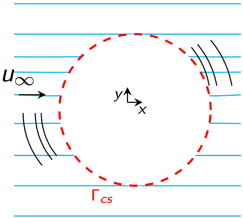
$$\frac{\partial}{\partial t}\left(\frac{\rho_0}{c_0^2}\frac{D_0\phi}{Dt}\right) - \nabla\cdot\left(\rho_0\nabla\phi - \frac{\rho_0}{c_0^2}\frac{D_0\phi}{Dt}\mathbf{u}_0\right) = 0 \quad (1)$$

2) FEM solution mapped on a **closed surface** Γ_{cs} in a uniform flow: ϕ, $\partial\phi/\partial n$

3) Far field radiation by a **integral formulation** (Wu et al., 1994):

$$\phi(\zeta) = \int_{\Gamma_{cs}} \frac{\partial\phi}{\partial n}G_a - \phi\frac{\partial G_a}{\partial n} - 2ikM_\infty G_a\phi n_x d\Gamma - \int_{\Gamma_{cs}} M_\infty^2\left(G_a\frac{\partial\phi}{\partial x} - \phi\frac{\partial G_a}{\partial x}\right)n_x d\Gamma \quad (2)$$





Further details on the method

Scattering and refraction on a non-uniform flow:

- Variational formulation of the full potential linearized wave equation

$$\int_{\Omega} -\frac{\rho_0}{c_0^2}\frac{D_0^* w}{Dt}\frac{D_0\hat{\phi}}{Dt} + \rho_0 \nabla w^* \cdot \nabla \hat{\phi} dV = \int_{\Gamma}\left[-\frac{\rho_0}{c_0^2} w^* \frac{D_0\hat{\phi}}{Dt}(\mathbf{u}_0 \cdot \mathbf{n}) + \rho_0 w^* \nabla \hat{\phi} \cdot \mathbf{n}\right] d\Gamma \quad (3)$$

Problem solved in the frequency domain $\hat{\phi} = \phi e^{i\omega t}$

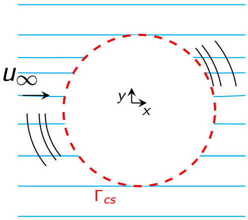
Convection on a uniform flow:

- Integral formulation

$$\phi(\boldsymbol{\zeta}) = \int_{\Gamma_{cs}} \frac{\partial \phi(\boldsymbol{\eta})}{\partial n_\eta} G_a(\boldsymbol{\zeta},\boldsymbol{\eta}) - \phi(\boldsymbol{\eta}) \frac{\partial G_a(\boldsymbol{\zeta},\boldsymbol{\eta})}{\partial n_\eta}$$
$$- 2ikM_\infty G_a(\boldsymbol{\zeta},\boldsymbol{\eta})\phi(\boldsymbol{\eta}) n_{\eta,x} - M_\infty^2 \left(G_a(\boldsymbol{\zeta},\boldsymbol{\eta}) \frac{\partial \phi(\boldsymbol{\eta})}{\partial x} - \phi(\boldsymbol{\eta}) \frac{\partial G_a(\boldsymbol{\zeta},\boldsymbol{\eta})}{\partial x} \right) n_{\eta,x} d\Gamma \quad (4)$$

Critical points of the method

- **Integral formulation** only exact with a uniform flow



- Acoustic particle velocity $\partial\phi/\partial n$, derived variable from FE solution, used as primary variable in the integral solution.

Error analysis

- H^1 error, FE solution (Babuska et al., 2007):

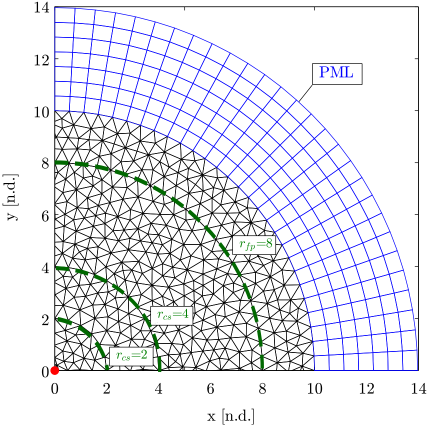
$$E \leq \underbrace{C_1 \left(\frac{kh}{2P}\right)^P}_{discretization\ \ error} + \underbrace{C_2 kL \left(\frac{kh}{2P}\right)^{2P}}_{dispersion\ \ error\ \ and\ \ pollution\ \ effect} \quad (5)$$

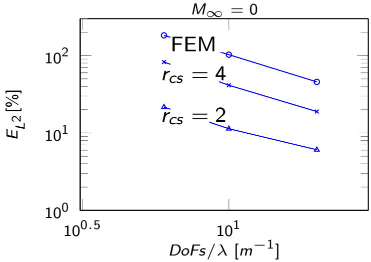
- Mach dependence: $C_2 \sim (1 - M)$ (Beriot et al., 2013)

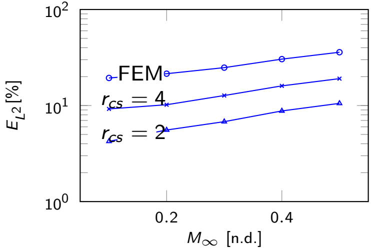
- Limiting pollution effects by means of the **integral formulation**.

Validation of the approach

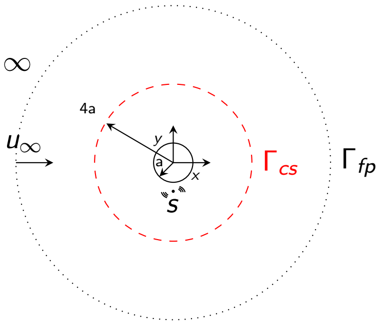
Convection of a monopole on a uniform flow:







Test case: scattering by a rigid cylinder



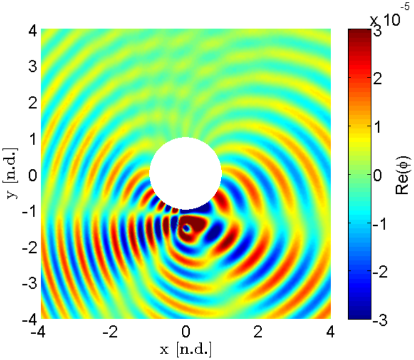
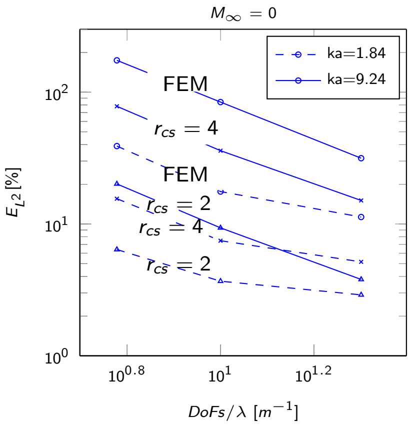
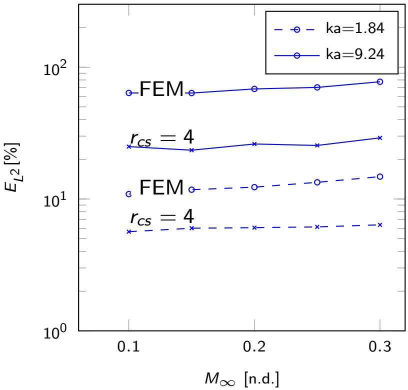


Figure: FEM $ka = 9.24$, $M_\infty = 0.3$

Error analysis on the test case

$L^2 - norm$ error at $r_{fp} = 8a$:





Concluding remarks

Model

- FE/integral-formulation solution for noise radiation with a mean flow
    - A validated FE approach was integrated with an existing integral formulation in the physical space for noise radiation in an unbounded domain with mean flow.

Insight

- The hybrid FEM/Radiating-surface approach limits the pollution effect even with a mean flow
- Limitations:
    - Integral formulation exact only on a uniform flow domain
    - Error dependent on the accuracy of the prediction of the acoustic particle velocity

CRANE project

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