

# A combined FEM/Radiating-surface approach for noise propagation in unbounded domains with mean flow

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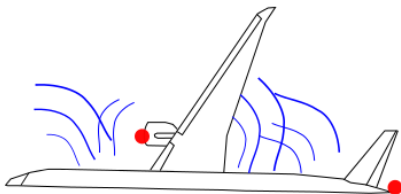
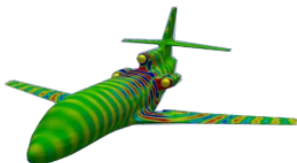
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## Motivation:

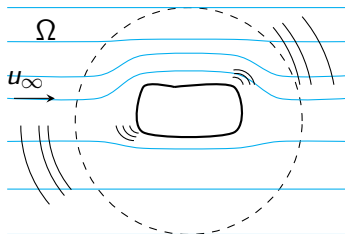
- Prediction of **noise radiation** for design and certification
- Impact of **non-uniform mean flow** on noise propagation
- Computational cost of far-field noise prediction with mean flow



- Background
- Numerical method
- Error analysis
- Numerical results
- Concluding remarks

## Why a hybrid method for noise propagation with mean flow?

- FE solutions suffer from **dispersion error** and **pollution effects**
- Integral formulation inherently satisfies **Sommerfeld condition**
- Complex **boundary** and **transmission conditions** when **Lorentz transformation** is applied to the integral formulation with mean flow.



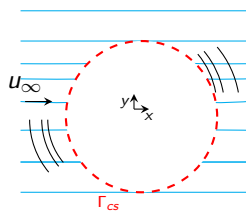
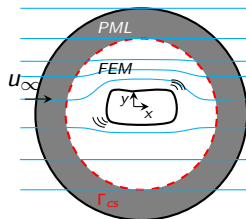
1) **Near field: FEM + PML** for scattering and refraction on a non-uniform flow

$$\frac{\partial}{\partial t} \left( \frac{\rho_0}{c_0^2} \frac{D_0 \phi}{Dt} \right) - \nabla \cdot \left( \rho_0 \nabla \phi - \frac{\rho_0}{c_0^2} \frac{D_0 \phi}{Dt} \mathbf{u}_0 \right) = 0 \quad (1)$$

2) FEM solution mapped on a **closed surface**  $\Gamma_{CS}$  in a uniform flow:  $\phi, \partial\phi/\partial n$

3) **Far field radiation** by a **integral formulation** (Wu et al., 1994):

$$\begin{aligned} \phi(\zeta) = & \int_{\Gamma_{CS}} \frac{\partial \phi}{\partial n} G_a - \phi \frac{\partial G_a}{\partial n} - 2ikM_\infty G_a \phi n_x d\Gamma \\ & - \int_{\Gamma_{CS}} M_\infty^2 \left( G_a \frac{\partial \phi}{\partial x} - \phi \frac{\partial G_a}{\partial x} \right) n_x d\Gamma \end{aligned} \quad (2)$$



## Scattering and refraction on a non-uniform flow:

- Variational formulation of the full potential linearized wave equation

$$\int_{\Omega} -\frac{\rho_0}{c_0^2} \frac{D_0^* w}{Dt} \frac{D_0 \hat{\phi}}{Dt} + \rho_0 \nabla w^* \cdot \nabla \hat{\phi} dV = \int_{\Gamma} \left[ -\frac{\rho_0}{c_0^2} w^* \frac{D_0 \hat{\phi}}{Dt} (\mathbf{u}_0 \cdot \mathbf{n}) + \rho_0 w^* \nabla \hat{\phi} \cdot \mathbf{n} \right] d\Gamma \quad (3)$$

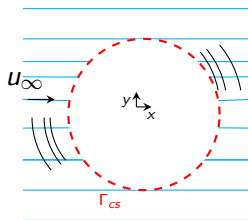
Problem solved in the **frequency domain**  $\hat{\phi} = \phi e^{i\omega t}$

## Convection on a uniform flow:

- Integral formulation

$$\begin{aligned} \phi(\zeta) = & \int_{\Gamma_{cs}} \frac{\partial \phi(\eta)}{\partial n_{\eta}} G_a(\zeta, \eta) - \phi(\eta) \frac{\partial G_a(\zeta, \eta)}{\partial n_{\eta}} \\ & - 2ikM_{\infty} G_a(\zeta, \eta) \phi(\eta) n_{\eta,x} - M_{\infty}^2 \left( G_a(\zeta, \eta) \frac{\partial \phi(\eta)}{\partial x} - \phi(\eta) \frac{\partial G_a(\zeta, \eta)}{\partial x} \right) n_{\eta,x} d\Gamma \end{aligned} \quad (4)$$

- **Integral formulation** only exact with a **uniform flow**



- **Acoustic particle velocity**  $\partial\phi/\partial n$ , derived variable from FE solution, used as primary variable in the integral solution.

- $H^1$  error, FE solution (Babuska et al., 2007):

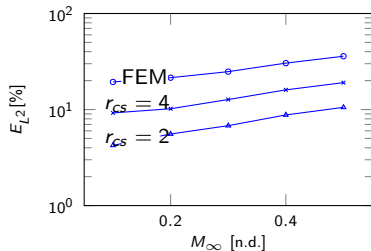
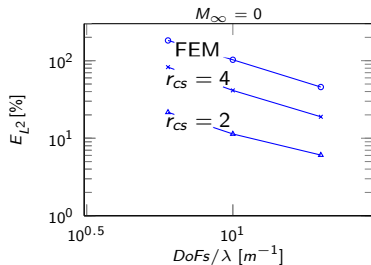
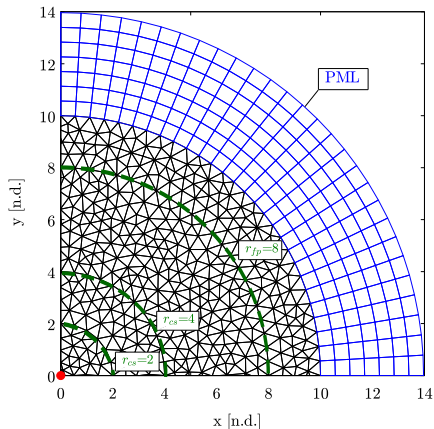
$$E \leq \underbrace{C_1 \left( \frac{kh}{2P} \right)^P}_{\text{discretization error}} + \underbrace{C_2 kL \left( \frac{kh}{2P} \right)^{2P}}_{\text{dispersion error and pollution effect}} \quad (5)$$

- Mach dependence:  $C_2 \sim (1 - M)$  (Beriot et al., 2013)
- Limiting pollution effects by means of the **integral formulation**.



# Validation of the approach

Convection of a monopole on a uniform flow:



# Test case: scattering by a rigid cylinder

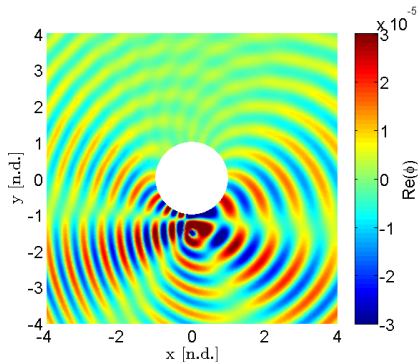
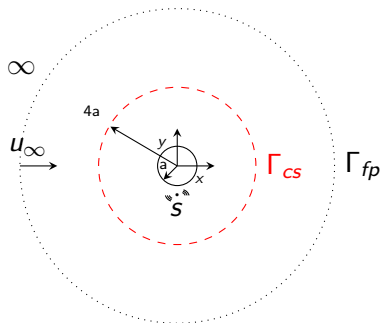
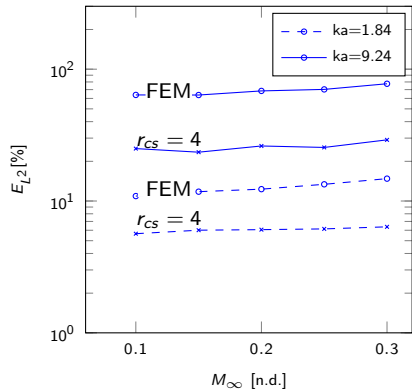
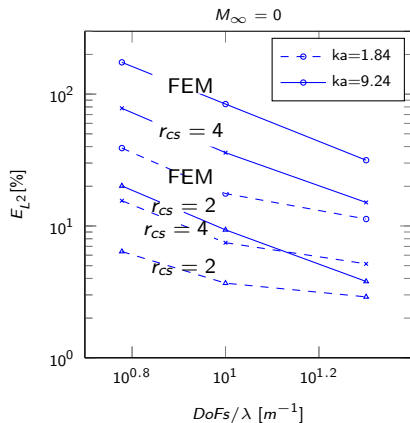


Figure: FEM  $ka = 9.24$ ,  $M_\infty = 0.3$

# Error analysis on the test case

$L^2$  - norm error at  $r_{fp} = 8a$ :



## Model

- FE/integral-formulation solution for noise radiation with a mean flow
  - ▶ A validated FE approach was integrated with an existing integral formulation in the physical space for noise radiation in an unbounded domain with mean flow.

## Insight

- The hybrid FEM/Radiating-surface approach limits the pollution effect even with a mean flow
- Limitations:
  - ▶ Integral formulation exact only on a uniform flow domain
  - ▶ Error dependent on the accuracy of the prediction of the acoustic particle velocity

CRANE project

[www.crane-eid.eu](http://www.crane-eid.eu)

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