Broadband noise for rotating blades: analysis of acceleration effects in the time and frequency domains

S. Sinayoko*

Institute of Sound and Vibration Research, University of Southampton

The importance of acceleration in predicting broadband aerofoil noise for high speed blades remains poorly understood. For high speed blades, acceleration becomes significant and Amiet's popular model predicting broadband noise for rotating blades should become invalid. However, previous work suggests that Amiet's model may be applicable even to high speed rotating blades. Previous attempts at clarifying this issue have focused on modelling trailing ede noise in the frequency domain. However, this issue is best studied in the time domain and for the leading edge problem. This paper implements Amiet's blade response functions for leading edge noise in the time domain, using a convected form of the Ffowcs-Williams and Hawkings equations. This allows to easily pinpoint the contribution of acceleration on aerofoil broadband noise. Results show that acceleration can be neglected for high speed propellers when the ratio of frequency over rotor angular velocity is larger than ten percent.

Introduction

Reducing aerofoil broadband noise is important for a wide range of applications, including open rotors, helicopters, wind turbines and cooling fans. Broadband noise can be decomposed into trialing edge noise and leading edge noise. Trailing edge noise is a type of self-noise due to the interaction of the boundary layer with the trailing edge of the blade. It is thought to be the main noise source in wind turbines. Leading edge noise is due to the interaction of turbulent structures with the leading edge of the blade. It is one of the main sources of airframe noise. Furthemore, it is of particular concern for high speed propellers and contra-rotating open rotors.

Trailing edge noise and leading edge noise can modelled by means of a distribution of dipoles representing loads over the blade surface. These dipoles are induced by the scattering of turbulent gusts at the leading or

*Research Fellow (Royal Commission for the Exhibition of 1851), ISVR, University of Southampton

trailing edge. In the case of a flat plate, this scattering process can be represented using an analytic blade response function,[1,2] that can be used to relate the wavenumber spectrum of the incoming turbulence to the sound pressure level in the far field. This approach, pioneered by Amiet and reviewd by Roger et al[3] provides an efficient way of predicting aerofoil broadband noise.

Amiet extended his method to rotating blades in uniform flows, by transforming the rotating blade problem into the stationary blade problem.[4] This transformation is obtained by modelling the rotation as a series of translations. Each translation can be modelled with the original stationary blade theory by working in the reference frame of the blade and using appropriate Doppler factors.[5]

Amiet's model for rotating aerfoils neglects the effect of acceleration due to the blade rotation. The effect of acceleration becomes significant when the rotational speed of the blade Ω is large compared to the angular frequency ω of the noise source.[6,7] This would be the case for blades rotating at high speed such as propellers or contra rotating open rotors. Nevertheless, Amiet's method is currently being used successfully to predict broadband noise in these applications.[8]

Blandeau et al[9] and Sinayoko et al[10] have attempted to study the range of validity of Amiet's approach to rotating blades, in the case of trailing edge noise. Blandeau et al found that Amiet's model was applicable in a certain frequency range. Sinayoko et al obtained a different result and found that Amiet's model was appicable even to high speed propeller blades. They explained that the effect of acceleration, although significant for the instantaneous spectrum, is negligible on average. Thus, although Amiet's model may under-predict or over-predict the instantaenous sound pressure level, it is very accurate for the time averaged sound pressure level.

However, both Blandeau et al's[9] and Sinayoko et al's[5] studies were based on new frequency-domain formulations that took into account the effect of acceleration but that were not validated independently. Furthemore, these new formulations are difficult to compare with Amiet's simplified model: they take the form of modal decompositions whereas Amiet's model takes the form of a simple integral. It is unclear how to identify the effect of acceleration in the new formulations. Thirdly, Amiet's theory for trailing edge noise is only valid at high frequencies relative to the chord. This condition can be violated for high speed rotating blades because of a strong Doppler shift: for certain angles around the hub, the source frequency can become very small compared to the observer frequency. It therefore remains unclear what the range of validity of Amiet's theory is for high speed blades.

One way to address the low frequency problem is to focus on leading edge noise. For leading edge noise, there is no frequency constraint relative to the blade chord. Furthermore, understanding the effect of acceleration is most easily done in the time domain.[6] Using the Ffowcs-Williams and Hawkings equations,[11] loading noise is split into two terms: the first one is due to the dipoles' unsteadiness, and corresponds to

Amiet's model; the second one is due to the acceleration of the blade and is the term neglected by Amiet. Thus, in this paper, we will implement Amiet's method in the time domain to explore the effect of acceleration on broadband noise. Leading edge noise will be our main focus.

Casper and Farassat have implemented Amiet's method in the time domain by using the Ffowcs-Williams and Hawkings[11] equations in combination with Amiet's blade response function for trailing edge noise,[12] or leading edge noise.[12] Their work was restricted to stationary airfoils but was applied to trailing edge noise radiation from rotating wind turbine blades by Lee et al.[13] A similar approach was proposed by Glegg et al,[14] who developed a compact source model for leading edge noise, for rotating airfoils. This work will follow the method presented in these papers to quantify the effect of acceleration on broadband noise for high speed blades. At high speed, convection effects become significant, so we will use the convected Ffowcs-Williams and Hawkings equations derived by Najafi-Yazdi et al.[15]

The structure of the paper is as follows. The frequency domain formulations of Amiet[4,5,7] and Sinayoko et al,[5] for both isolated and rotating airfoils, are summarized in section §I. Section II focuses with the time domain formulation of Amiet's method. Section III presents some preliminary results and discussion. The final version of this paper will present an extensive parametric study and will quantify the effect of acceleration on broadband noise for high speed propellers.

I. Frequency domain formulations

A. Amiet's broadband noise theory for isolated blades

Consider a flat plate in a uniform flow of Mach number M_X at zero angle of attack. The observer location is expressed using a cartesian coordinate system (X, Y, Z), with X in the chordwise direction and pointing downstream and Z in the vertical direction. According to Amiet,[2,16] the PSD at frequency ω is of the form

$$S_{pp}(\omega) = \left(\frac{\omega}{c_0}\frac{Z}{4\pi\sigma^2}\right)^2 S_{ff}(\omega), \tag{1}$$

where S_{ff} is a frequency force power spectral density,

$$\sigma^2 = X^2 + \beta^2(Y^2 + Z^2), \qquad \beta^2 = 1 - M_X^2. \tag{2}$$

For a stationary flat plate in a uniform flow,[1]

$$S_{ff}(\omega) = 2\pi^3\rho^2 SC^2|\Psi_{LE}(k_X, k_S, k_C)|^2\Phi_{ww}(k_X, k_S)S_{qq}(\omega), \tag{3}$$

where S and C denote the blade span and chord,

$$k_X = \frac{\omega}{U_c}, \qquad k_S = \frac{\omega}{c_0}\frac{Y}{\sigma}, \qquad k_C = \frac{\omega}{c_0\beta^2}\left(M_X - \frac{X}{\sigma}\right), \tag{4}$$

the leading edge noise acoustic lift Ψ_{LE} is defined by Amiet in equations (31) to (33) of Ref,[7] and the inflow wavenumber spectrum is given by

$$\Phi_{ww}(k_X, k_S) = \frac{4}{9\pi}\frac{u_{\mathrm{rms}}^2}{k_e^2}\frac{\hat{k}_x^2 + \hat{k}_S^2}{(1+\hat{k}_x^2+\hat{k}_S^2)^{7/3}}, \tag{5}$$

where $k_e = \sqrt{\pi} Gamma(5/6)/(L\Gamma(1/3))$, L is the integral length scale of the inflow turbulence, and u_{rms} is computed from the turbulence intensity.

B. Amiet's broadband noise theory for rotating blades

Instantaneous power spectral density Following Schlinker and Amiet's approach,[4] the instantaneous PSD is given in source time τ as

$$S_{pp}^{(A)}(\mathbf{x_o}, \tau, \omega) = \left(\frac{\omega'}{\omega}\right)^2 S'_{pp}(\mathbf{X}, \tau, \omega'), \tag{6}$$

where ω' and S'_{pp} are the frequency and the instantaneous power spectral density in the reference frame of the source respectively. The observer position $\mathbf{X}$ is defined as

$$\mathbf{X} = \underline{\mathbf{R_y}}(\alpha)\underline{\mathbf{R_z}}(\pi/2-\alpha)(\mathbf{x_o} - \mathbf{x_p}), \tag{7}$$

where α is the pitch angle, $\mathbf{x_p} \approx \mathbf{M_{BO}c_0T_e}$ is the present source position (assuming that the source is emitted at the hub, which is valid for an observer in the far field) and is expressed in terms of the blade Mach number $\mathbf{M_{BO}} = M_t\hat{\boldsymbol{\gamma}}$ relative to the observer. $\underline{\mathbf{R_z}}$ and $\underline{\mathbf{R_y}}$ denote the rotation matrices about the z-axis and y'-axis respectively, with $y' = \underline{\mathbf{R_z}}(\pi/2-\alpha)\mathbf{y}$. The propagation time T_e is obtained from $R_e \equiv c_0T_e$, where R_e is the distance from the convected (or retarded) source position to the observer location:

$$R_e = \frac{R\left(-M_z\cos\Theta + \sqrt{1-M_z^2\sin^2\Theta}\right)}{1-M_z^2}, \quad \text{(far field)}, \tag{8}$$

where $\Theta = \pi - \theta$ denotes the angle between the flow Mach number relative to the observer and $\mathbf{x_o}$ (see figure 1).

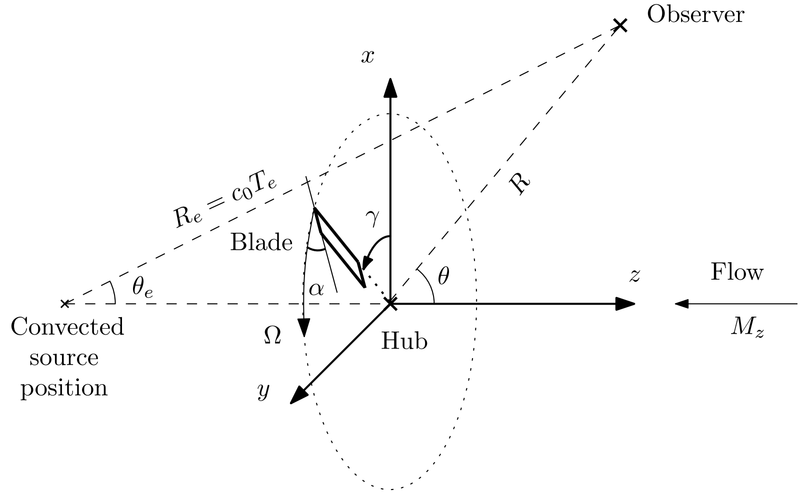


Figure 1: Stationary observer in a uniform flow. The reference frame is attached to the hub around which a blade element is rotating at angular velocity Ω. For an obsever in the far field, the source can be assumed to be located at the hub so the convected source position is as located downstream of the hub at $\mathbf{x_c} = -M_z c_0 T_e$.

Finally, the source frequency ω' is related to the observer frequency ω through the Doppler shift[17]

$$\frac{\omega}{\omega'} = 1 + \frac{\mathbf{M_{BO}} \cdot \widehat{\mathbf{CO}}}{1 + (\mathbf{M_{FO}} - \mathbf{M_{BO}}) \cdot \widehat{\mathbf{CO}}} \quad \text{(far field)}, \tag{9}$$

where $\mathbf{M_{FO}} = -M_z \hat{\boldsymbol{z}}$ is the flow Mach number relative to the observer, and $\widehat{\mathbf{CO}} = \mathbf{CO}/|\mathbf{CO}|$ is the unit vector from the convected source position to the observer position (see figure 1).

The spectrum S'_{pp} in the reference frame of the blade can be computed from equation (1), using a frequency force spectrum S_{ff} representative of a distributed source from equation (3).

Time averaged power spectral density The time averaged PSD is obtained by averaging equation (6) over τ, giving

$$\overline{S}^{(A)}_{pp}(\mathbf{x_o}, \omega) = \frac{1}{T} \int_0^T \left(\frac{\omega'}{\omega} \right)^2 S'_{pp}(\mathbf{X}, \tau, \omega') \, \mathrm{d}\tau. \tag{10}$$

II. Time domain formulations

A. Ffowcs-Williams and Hawkings equation

The pressure due to loading noise for a moving blade in a uniform flow can be expressed as follows, using the modified Ffowcs-Williams and Hawkings equation due to Najafi-Yazdi et al:[15]

$$p = \frac{1}{c_0}\int_{f=0}\left[\frac{\dot{L}_i\tilde{R}_i + L_i\dot{\tilde{R}}_i}{R^*(1-M_R)^2}\right]\mathrm{d}\eta - \frac{1}{c_0}\int_{f=0}\left[\frac{\dot{R}^* L_i\tilde{R}_i}{R^{*2}(1-M_R)^2}\right]\mathrm{d}\eta + \frac{1}{c_0}\int_{f=0}\left[\frac{\dot{M}_R L_i\tilde{R}_i}{R^*(1-M_R)^3}\right]\mathrm{d}\eta + \int_{f=0}\left[\frac{L_i\tilde{R}_i^*}{R^{*2}(1-M_R)}\right]\mathrm{d}\eta. \quad (11)$$

where the terms in square brackets are evaluated at the retarded time, the dots represent partial time derivatives with respect to retarded time, and

$$R^* = (z_o - z_s)^2 + \beta^2(x_o - x_s)^2 + \beta^2(y_o - y_s)^2, \quad (12)$$

$$\tilde{\mathbf{R}} = \frac{x_o - x_s}{R^*}\hat{\mathbf{x}} + \frac{y_o - y_s}{R^*}\hat{\mathbf{y}} + \frac{1}{\beta^2}\left(M_z + \frac{z_o - z_s}{R^*}\right)\hat{\mathbf{z}}, \quad (13)$$

$$\tilde{\mathbf{R}}^* = \beta^2\frac{x_o - x_s}{R^*}\hat{\mathbf{x}} + \beta^2\frac{y_o - y_s}{R^*}\hat{\mathbf{y}} + \frac{z_o - z_s}{R^*}\hat{\mathbf{z}}, \quad (14)$$

$$M_R = \mathbf{M_{SO}}\cdot\tilde{R}. \quad (15)$$

In the far field, terms of order $1/R^{*2}$ can be neglected. Furthermore, since $\dot{\tilde{R}}_i$ varies with R^* in the far field, terms of the form $\dot{\tilde{R}}_i/R^*$ can also be neglected. Furthermore, L_i can be expressed in terms of the pressure jump as $\Delta p\, n_i$, where n_i is the vector normal to the blade. Thus, in the far field, the second and fourth terms in equation (11) can be neglected and loading noise takes the form

$$p \approx \frac{1}{c_0}\int_{f=0}\left[\frac{\Delta\dot{p}n_i\tilde{R}_i}{R^*(1-M_R)^2}\right]\mathrm{d}\eta + \left\{\frac{1}{c_0}\int_{f=0}\left[\frac{\Delta p\dot{n}_i\tilde{R}_i}{R^*(1-M_R)^2}\right]\mathrm{d}\eta - \frac{1}{c_0}\int_{f=0}\left[\frac{\dot{M}_R\Delta p n_i\tilde{R}_i}{R^*(1-M_R)^3}\right]\mathrm{d}\eta\right\}. \quad (16)$$

The first term in the above equation is due to unsteady loading. The second one, between curly brackets, is due to the acceleration of the blade. This second effect is neglected in Amiet's model for broadband noise for rotating blades. In the time domain, Amiet's prediction should be equivalent to the noise due to unsteady loading only, p_{ul}, where

$$p_{\mathrm{ul}} \equiv \frac{1}{c_0}\int_{f=0}\left[\frac{\Delta\dot{p}n_i\tilde{R}_i}{R^*(1-M_R)^2}\right]\mathrm{d}\eta. \quad (17)$$

B. Blade loading

Given the wavenumber power spectral density (PSD) $\Phi_{ww}(k_X, k_y)$ of the incident velocity fluctuations, we wish to define a gust amplitude distribution $\tilde{p}_0(k_x, k_y)$ such that the associated PSD matches Φ_{ww}. This can be done by expressing the power $\Delta\mathcal{P}$ contained in the incident wall-pressure signal within a wavenumber bandwidth $(\Delta k_X, \Delta k_y)$ using either $\Delta\tilde{p}_0$ or the two-sided wavenumber PSD Φ_{ww}. The signal is of the form

$$\Delta p_0(x,y,t) = \Delta\tilde{p}_0(k_x,k_y)\cos(k_x(x-Ut)+k_y y-\omega t+\phi), \tag{18}$$

where ϕ is a random variable uniformely distributed in $[0,2\pi]$, so the power is given by

$$\Delta\mathcal{P} = \frac{1}{2}\Delta\tilde{p}_0^2(k_x,k_y). \tag{19}$$

On the other hand, the power can be expressed in terms of the PSD as

$$\Delta\mathcal{P} = 4\Phi_{ww}(k_x,k_y)\Delta k_x\Delta k_y, \tag{20}$$

where the factor of 4 accounts for the negative wavenumber components. Equating equations (19) and (20) yields

$$\tilde{p}_0(k_x,k_y) = \sqrt{8\Phi_{ww}(k_x,k_y)\Delta k_x\Delta k_y}. \tag{21}$$

Note that the above expression differs slightly from that derived by Casper and Farassat,[12] who derived a factor of π instead of $\sqrt{8}$. The author was unable to understand the presence of the factor of π in Ref.,[12] but the difference on the sound pressure level amounts to less than 1 dB.

Using Amiet's blade response function, we can now generate a stochastic signal for the pressure jump $\Delta p(x,y,t)$ as

$$\Delta p(x,y,t) = \mathrm{Re}\left\{\sum_{n=1}^{N}\sum_{m=1}^{M}\tilde{p}_0(k_{x,n},k_{y,m})e^{i(k_{x,n}(x-Ut)+k_{y,m}y-\omega_n t+\phi_n)}g(x,k_x,k_y)\Delta k_x\Delta k_y\right\}, \tag{22}$$

where the ω_n are a set of N angular frequencies and the ϕ_n are independent random variables uniformely distributed in $[0,2\pi]$, and the blade response function is given by[7]

$$g(x,k_x,k_y) = \frac{S(k_x^*)}{\pi\beta}\sqrt{\frac{1-\overline{x}}{1+\overline{x}}}\exp\left\{i[\mu M x + f(M_\infty k_x^*)]\right\}, \quad \text{if} \quad \overline{\mu}_\infty < 0.4 \tag{23}$$

$$g_2(x,k_x,k_y) = g_1(x,k_x,k_y)\sqrt{(1+\overline{x})/2}\left\{(1+i)E^*[2k_x^*M_\infty(1-\overline{x})]-1\right\}, \quad \text{if} \quad \overline{\mu}_\infty > 0.4, \tag{24}$$

$$\text{where} \quad g_1(x,k_x,k_y) = \exp(-i\Theta_4)/(\pi\sqrt{\pi k_x(1+M_\infty)(1+\overline{x})}), \tag{25}$$

where the overline means normalization by the the half-chord and

$$k_x^* = k_x/\beta^2, \qquad \mu = M k_x/\beta^2 \tag{26}$$

$$\mu_\infty = M_\infty k_{x\infty}/\beta_\infty \qquad M_\infty = \sqrt{M^2 - \beta^2 k_y^2/k_x^2}, \qquad \beta_\infty = \beta\sqrt{1 + k_y^2/k_x^2}. \tag{27}$$

For a given observer position, only the gusts travelling towards the observer contribute to the noise: the wavenumber vector must lie in the plane defined by the observer direction and the chordwise direction. For example, for a stationary aerofoil and an observer in the mid-chord plane, only incident waves parallel to the edge, i.e. such that $k_y = 0$ contribute to the noise at the observer location.[1,2,5] Given the specific value k_{y,m_o} of the spanwise wavenumber contributing to the far field noise for the problem at hand, we can express the incident pressure field as $\delta_{m,m_o} p_0(k_{x,n}, k_{y,m_o})$, using the Kronecker symbol, so equation (22) reduces to

$$\Delta p(x, y, t) = \mathrm{Re}\left\{\sum_{n=1}^{N} \tilde{p}_0(k_{x,n}, k_{y,m_o}) e^{i(k_{x,n}(x-Ut)+k_{y,m_0}y-\omega t+\phi_n)} g(x, k_{x,n}, k_{y,m_0}) \Delta k_x \Delta k_y\right\}. \tag{28}$$

Finally, for a blade element of span S, the spanwise wavenumber bandwidth is expressed as $\Delta k_y = 2\pi/S$.

C. Rotating blade

For a rotating blade element, the source frequency ω' is Doppler shifted relative to the observer frequency *omega*. The Doppler shift is defined in equation (9). Over one rotation around the hub, the source frequency oscillates between a minimum and a maximum value that depend on the Doppler shifts as a function of azimuthal angle. For a given observer frequency, we use a finite number of uniformely distributed frequency bins between those minimum and maximum values.

For each source emission time τ, we estimate the instantaneous pressure field by combining equation (28), with N=1 and $k_{x,n} = \omega'(\tau)/U$, and equation (16). We compute the time series over at least 4 periods, with at least 8 points per period. The period is relative to observer ω if $\omega > \Omega$, and relative to the blade angular velocity Ω if $\omega < \Omega$. We discretize the blade in the chordwise direction using at least 256 elements. The instantaneous PSD is obtained by taking the average of the square pressure. Finally, the averaged PSD is obtained by averaging the instantaneous PSD over al azimuthal angles.

III. Results and Discussion

The blade element paramters and operating conditions are listed in table 1. We compute the time averaged PSD for a variety of test cases using the procedure of section II.

In all cases, the integral length scale is set to half the blade element radial location. The turbulence intensity is taken to be 5 percent of the inflow velocity relative to the hub. The observer is located at 100 wavelengths to the hub, to satisfy the far field condition.

	radius	chord	M_{BO}	Pitch	M_{FO}	M_{FB}
Take-off	1.80	0.31 m	0.748	38 deg	0.228	0.782
Cruise	1.80	0.31 m	0.748	13 deg	0.584	0.949

Table 1: Propeller blade segment and flow paramters at cruise and take-off conditions.[9] The span is defined as a third of the radius. The wind Mach numbers, relative to the observer $M_{FO}(= M_z)$ or to the blade $M_{FB}(= M_X)$ can be obtained from the pitch angle and blade speed M_{BO} but are given for completeness.

A. Stationary blades

Figure 2 plots the sound pressure level estimated using the time-domain FW-H formulation for a stationary propeller blade element in a uniform flow of Mach 0.782 (take-off, dashed line with red squares) or 0.949 (cruise, dashed line with black circles). The corresponding estimates based on Amiet's frequency domain formulation are shown in solid lines. The curves were computed using 750 elements along the blade chord, with a time series duration of 64 periods, with 32 samples per period, and assuming $k_y = 0$.

The curves obtained using the FW-H are in good agreement with Amiet's original formulation, which validates the procedure in section B. This suggests that the factor $\sqrt{8}$ in equation (21) is correct and should be preferred to the factor π employed by Casper and Farassat.[12]

B. Rotating blades

The SPL spectrum for an observer at angle $\pi/4$ to the z-axis, at 100 wavelengths from the hub, is shown in figure 3 for both take-off (top figure) and cruise conditions (bottom figure). The SPL is plotted using 4 different formulations: Amiet's frequency domain formulation (solid black line); the FW-H time domain formulation without acceleration effects (red dashed line with circles), the FW-H time domain formulation with acceleration terms only (green dashed line with triangles); the FW-H full time domain formulation (blue dashed line with squares). The normalized frequency ω/Ω varies between 10^{-2} and 10^{2}. The FW-H results were obtained using 4 periods with 8 samples per period, with a mesh size of 256 points, 10 frequency bins and assuming $k_y = 0$.

Simiarly, figure 4 presents the SPL directivity for the propeller blade element at take-off, at (a) low frequency $\omega/\Omega = 0.1$ and (b) high frequency $\omega/\Omega = 10$. The results were obtained using 750 segments along the chord, 8 periods and 256 samples per period. Only 9 angular values were computed to limit the computation time.

The red dashed line with circles in figure 3, which neglects the acceleration terms in equation (16), is in very good agreemnt with the solid black line from Amiet's frequency domain formulation. This validates the

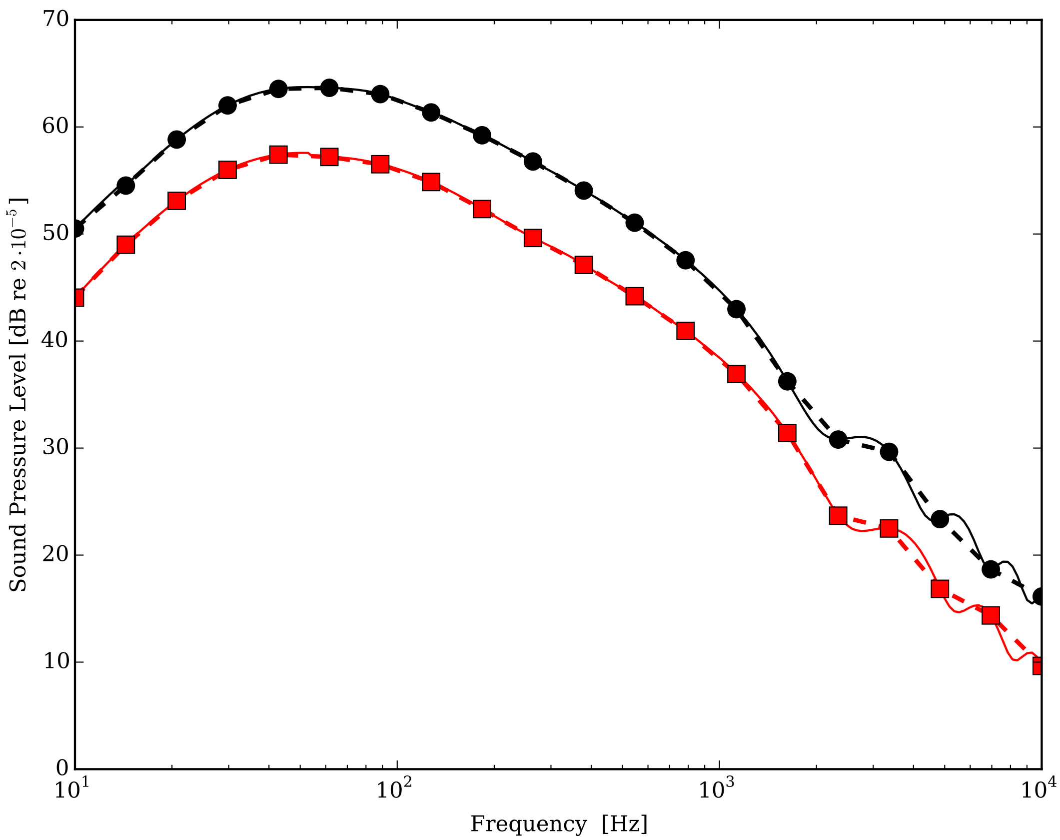


Figure 2: SPL for leading edge noise for stationary propeller blade elements at take-off (red squares) and cruise (black circles) conditions, for an observer at 100 wavelengths from the hub at angle $pi/4$ to the z-axis. Thin solid lines correspond to Amiet's frequency domain prediction from equations (1) and (10). The dashed lines with markers correspond to the time domain FW-H prediction.

approach used to estimate broadband noise in the time domain as described in section C.

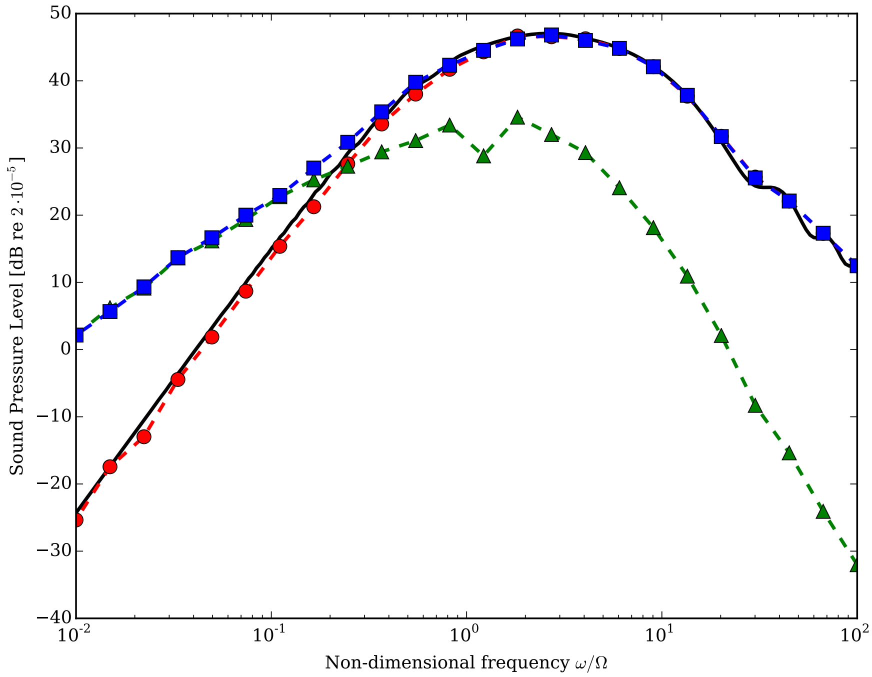
The green curve with triangles and the blue curve with square, showing respectively the SPL based on acceleration terms only and the SPL based on both acceleration term and unstead loading, indicate that acceleration terms become the dominant noise source at very low frequency $\omega/Omega \leq 0.2$. At higher frequencies, the SPL due to acceleration terms is at least 10 dB lower than that due to unstead loading: the effect of acceleration is negligible. Most importantly, Amiet's formulation is accurate even when $\omega \approx \Omega$, or for slightly lower frequencies. This is unexpected as Amiet's predicted that his model should apply when $\omega \gg \Omega$. This confirms the results of Sinayoko et al[10] for trailing edge noise. The reason for this discrepancy must be that acceleration terms, although higher than unsteady loading terms for $\omega \leq \Omega$, do not contribute to the time average PSD above lower frequencies of the order of $\Omega/10$: their contribution to the instantaneous PDS averages out to 0.

Figure 4 also confirms that acceleration effects are negligible at high frequencies (top figure) but dominate at low frequencies. It can be seen also that the directivity of acceleration terms and unstead loading terms are somewhat different, and that acceleration terms do not contribute to the average SPL for observers located along the z-axis (θ equal to 0 or π).

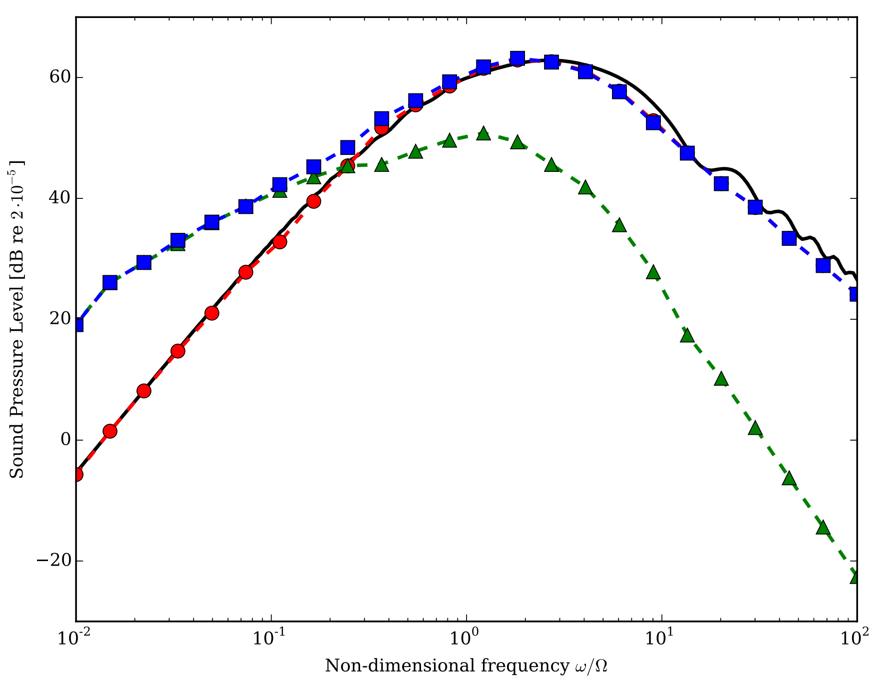
Finally, the agreement between Amiet's formulation and the FW-H formulation without acceleration terms is accurate to within 3 decibels. This may be due to the fact that the FW-H formulation does not appear to fully capture the ripples visible in Amiet's approach at high frequency in figure 3. Thus, although the FW-H formulation captures the slope in the spectrum correctly, as shown in figure 3, the amplitude may be slightly off by a few dB. However, this effect appears less significant at low frequencies, as shown in figure 4(a), which is the more important frequency range in this paper.

Conclusions

This paper presents for the first time a semi-analytical method for estimating leading edge noise for stationary or rotating aerofoils by combining Amiet's formulation with the FW-H equations. The paper confirms that acceleration effects are negligible at high frequencies compared to the rotor angular frequency. Interestingly, this remains true even for frequencies of the order of, or up to one order of magnitude smaller, than the rotor angular frequency. The reason is that acceleration terms may not contribute to the time averaged sound pressure level until very low frequencies. The main outcome of this paper is that Amiet's broadband noise formulation for rotating aerofoils may be used over the entire frequency range of interest for high speed propellers and open rotors.

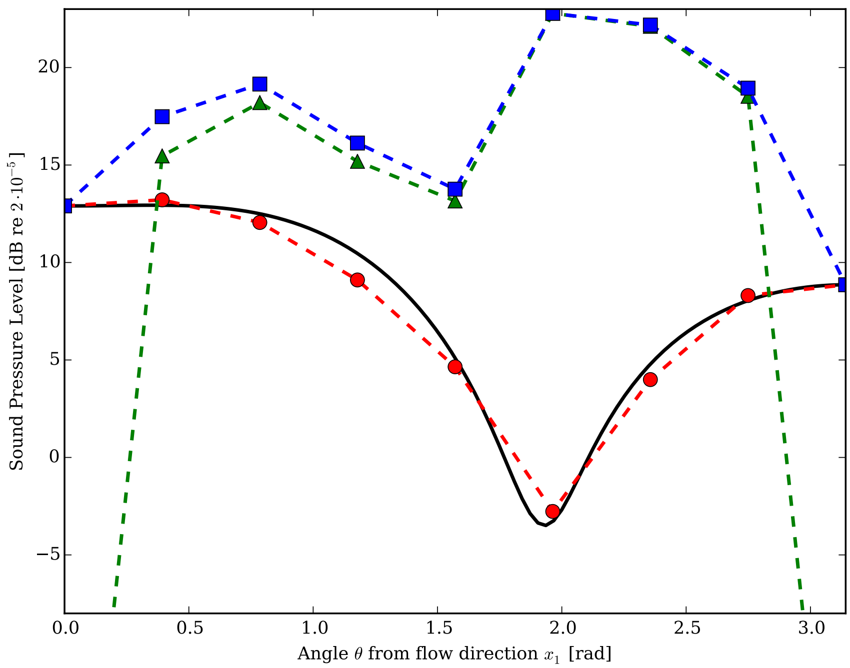


(a) Propeller blade element at take-off.

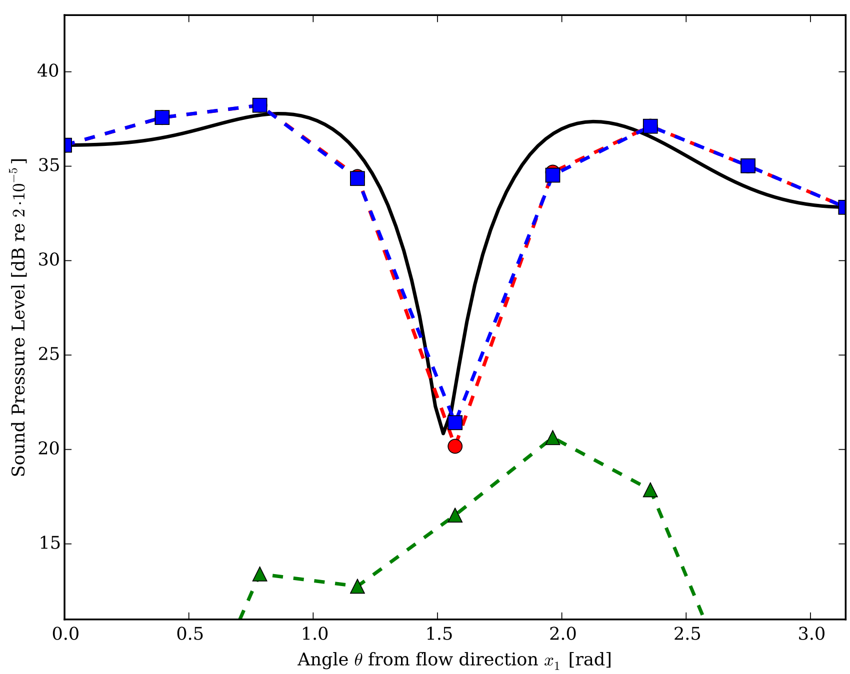


(b) Propeller blade element at cruise.

Figure 3: SPL for leading edge noise for a rotating propeller blade element at take-off (top) or cruise (bottom) conditions using the FW-H formulation (dashed lines with markers) or Amiet's formulation (black solid line). Three different versions of the FW-H formulation are plotted: unstead loading term (◯) , acceleration term (△), both (□). The acceleration term dominates when $\omega/\Omega < 0.2$.



(a) Low frequency: $\omega/\Omega = 0.1$



(b) High frequency: $\omega/\Omega = 10$

Figure 4: SPL directivity for leading edge noise for a rotating propeller blade element at take-off using the FW-H formulation (dashed lines with markers) or Amiet's formulation (black solid line). The observer is at 100 wavelengths from the hub. Three different versions of the FW-H formulation are plotted: unsteady loading term (◯) , acceleration term (△), both (□). The acceleration term dominates at low frequency.

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