

Trailing edge noise for rotating blades

Analysis and comparison of two classical approaches

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Motivation



Outline

Isolated airfoil theory

Amiet's theory for rotating airfoils

Kim-George's theory for rotating airfoils

Results

Conclusions

Review

Trailing edge for isolated airfoils

- ▶ Amiet 1974, 1975, 1976
- ▶ Roger and Moreau 2005, 2009

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Trailing edge for isolated airfoils

- ▶ Amiet 1974, 1975, 1976
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Trailing edge for rotating airfoils

- ▶ Amiet 1976
- ▶ Schlinker and Amiet 1981
- ▶ Kim and George 1982
- ▶ Blandeau and Joseph 2011

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Isolated airfoil theory

$$S_{pp} = \alpha \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

Isolated airfoil theory

► Amplitude

$$S_{pp} = \alpha$$

$$|\Psi|^2$$

$$l_y$$

$$S_{qq}$$

Isolated airfoil theory

► Amplitude

$$S_{pp} = \alpha$$

$$|\Psi|^2$$

$$l_y$$

$$S_{qq}$$

► Acoustic lift

Isolated airfoil theory

- ▶ Amplitude

$$S_{pp} = \alpha \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

- ▶ Acoustic lift
- ▶ Spanwise correlation length

Isolated airfoil theory

- ▶ Amplitude

$$S_{pp} = \alpha \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

- ▶ Acoustic lift
- ▶ Spanwise correlation length
- ▶ Surface spectrum

Outline

Isolated airfoil theory

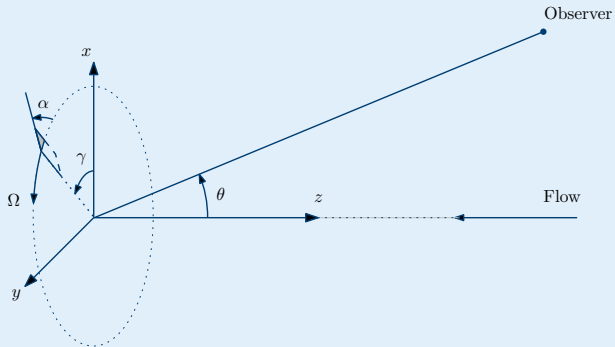
Amiet's theory for rotating airfoils

Kim-George's theory for rotating airfoils

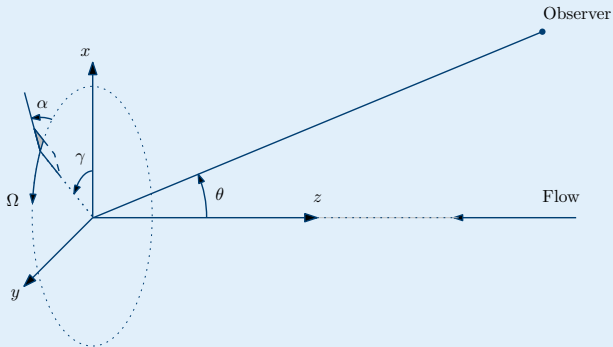
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Rotating airfoil

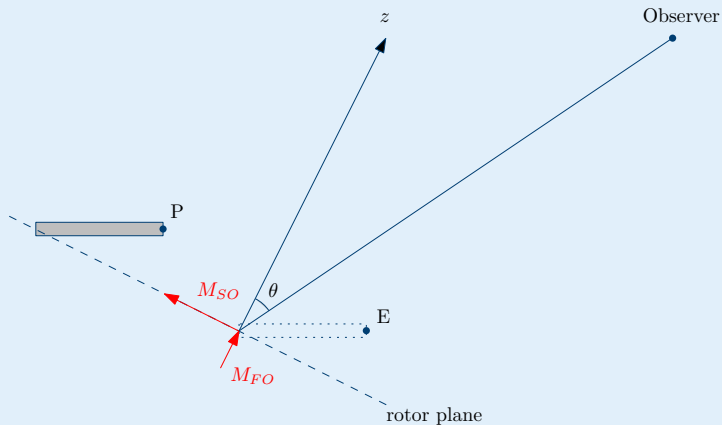


Rotating airfoil

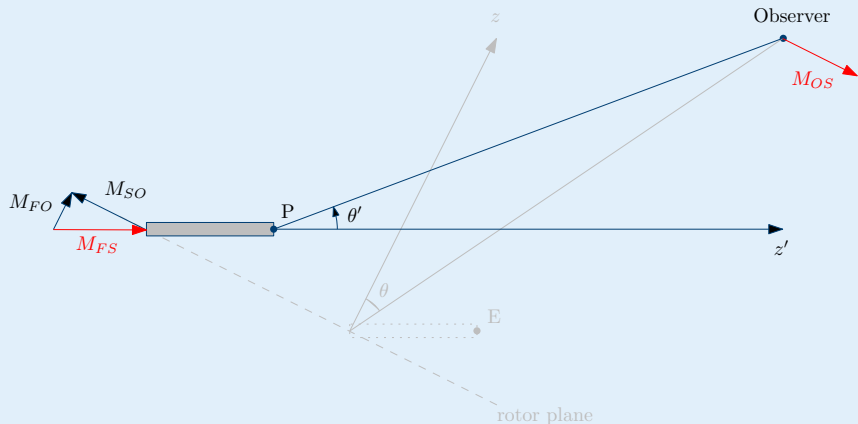


$$S_{pp}(\omega) = \int S_{pp}(\omega, \gamma) dt$$

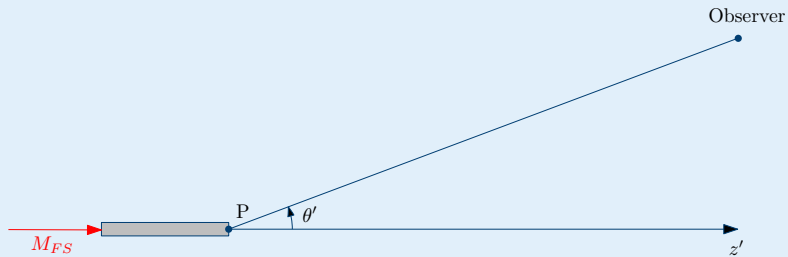
Change of reference frame



Change of reference frame



Change of reference frame



Moving observer in wind tunnel

- ▶ $p(\mathbf{x}, t)$: no effect
- ▶ $p(\mathbf{x}, \omega)$: Doppler shift $\omega' \rightarrow \omega$

Moving observer in wind tunnel

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PSD for moving observer

$$S_{pp}(\omega, \gamma) = \frac{\omega'}{\omega} S'_{pp}(\omega', \gamma)$$

Moving observer in wind tunnel

- ▶ $p(\mathbf{x}, t)$: no effect
- ▶ $p(\mathbf{x}, \omega)$: Doppler shift $\omega' \rightarrow \omega$

PSD for moving observer

$$S_{pp}(\omega, \gamma) = \frac{\omega'}{\omega} S'_{pp}(\omega', \gamma)$$

Time for moving observer

$$dt = \frac{\omega'}{\omega} dt'$$

Amiet's theory for rotating blade

$$S_{pp}(\omega) = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{\omega'}{\omega} \right)^2 S'_{pp}(\omega', \gamma) d\gamma$$

But... not everyone agrees

$$S_{pp}(\omega) = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{\omega'}{\omega} \right)^e S'_{pp}(\omega', \gamma) d\gamma$$

Source	Exponent e
Amiet (1976)	1
Rozenberg et al (2010)	1
Schlinker and Amiet (1981)	2
Blandeau and Joseph (2011)	-2

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Isolated airfoil theory

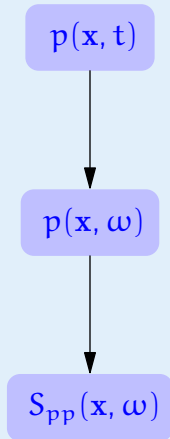
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Kim-George approach



Kim-George approach

$$S_{pp} = \sum_m \text{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

Kim-George approach

► Amplitude

$$S_{pp} = \sum_m \text{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$$
The diagram illustrates the Kim-George approach to calculating the amplitude S_{pp} . It shows the equation $S_{pp} = \sum_m \text{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$. The term B is highlighted in a purple box, and an arrow points from the word "Amplitude" to it. The other terms are highlighted in colored boxes: $|\Psi|^2$ in a red box, l_y in a green box, and S_{qq} in a yellow box.

Kim-George approach

▶ Amplitude

$$S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$$
The equation $S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$ is shown with several elements highlighted in colored boxes: \mathbf{B} is in a purple box, $|\Psi|^2$ is in a red box, l_y is in a green box, and S_{qq} is in a yellow box. Two blue arrows point from text labels to these boxes: one from 'Amplitude' to the purple box, and one from 'Acoustic lift' to the red box.

▶ Acoustic lift

Kim-George approach

- ▶ Amplitude

$$S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

The diagram shows the equation $S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$ with four colored boxes around the terms: a purple box around \mathbf{B} , a red box around $|\Psi|^2$, a green box around l_y , and a yellow box around S_{qq} . Three arrows point from text labels to these boxes: 'Amplitude' points to \mathbf{B} , 'Acoustic lift' points to $|\Psi|^2$, and 'Spanwise correlation length' points to l_y .

- ▶ Acoustic lift
- ▶ Spanwise correlation length

Kim-George approach

- ▶ Amplitude

$$S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$$

The diagram illustrates the Kim-George approach to calculating the pressure spectrum S_{pp} . The equation is $S_{pp} = \sum_m \mathbf{B} \quad |\Psi|^2 \quad l_y \quad S_{qq}$. Each term in the product is enclosed in a colored box: \mathbf{B} is purple, $|\Psi|^2$ is red, l_y is green, and S_{qq} is yellow. Arrows from the text below point to these boxes: 'Amplitude' points to \mathbf{B} , 'Acoustic lift' points to $|\Psi|^2$, 'Spanwise correlation length' points to l_y , and 'Surface spectrum' points to S_{qq} .

- ▶ Acoustic lift
- ▶ Spanwise correlation length
- ▶ Surface spectrum

Summary

Amiet:

$$S_{pp} = \int A |\Psi|^2 l_y S_{qq} d\gamma$$

Kim-George:

$$S_{pp} = \sum_m B |\Psi|^2 l_y S_{qq}$$

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Isolated airfoil theory

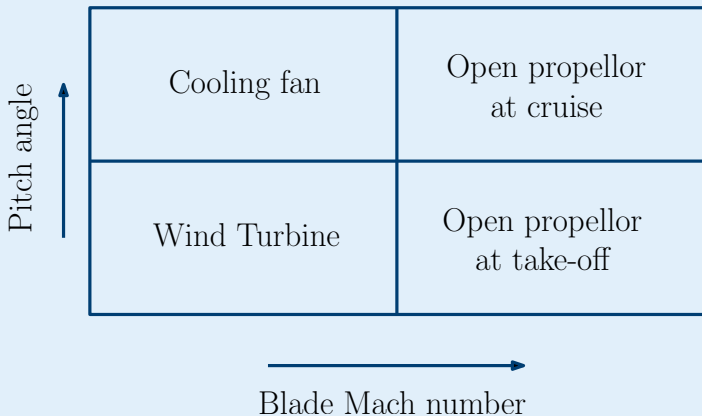
Amiet's theory for rotating airfoils

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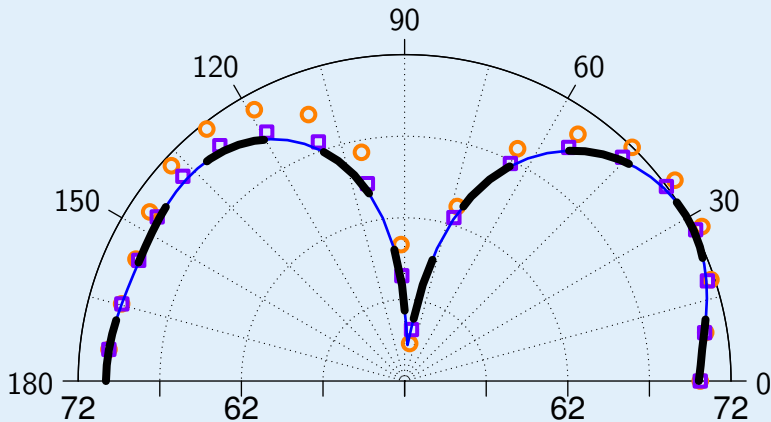
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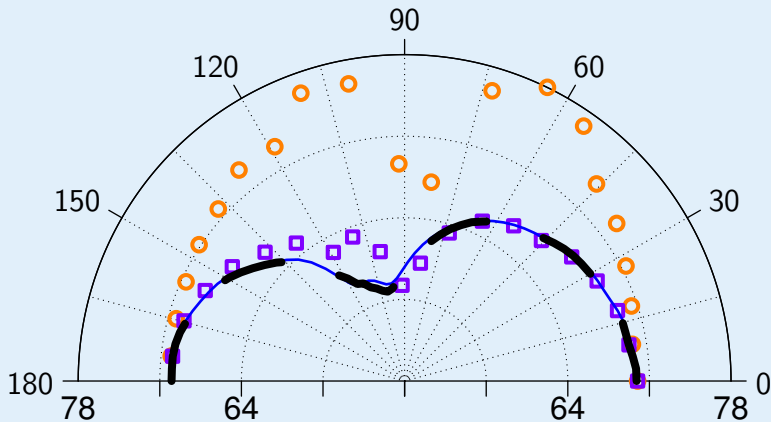
Blade elements



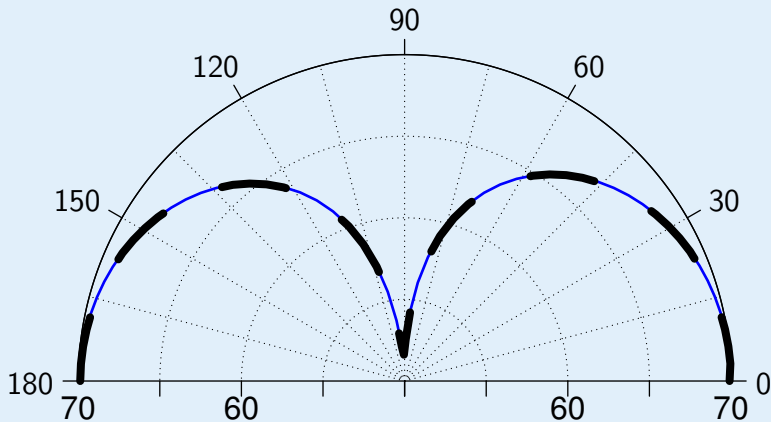
Wind turbine $k_c = 5$



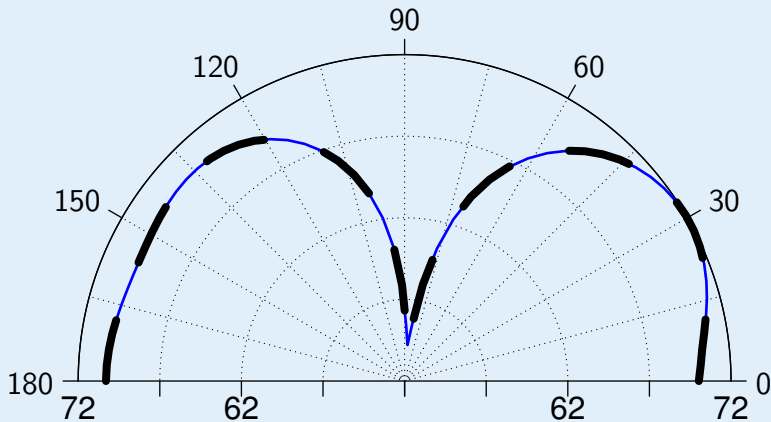
Open propellor takeoff $k_c = 5$



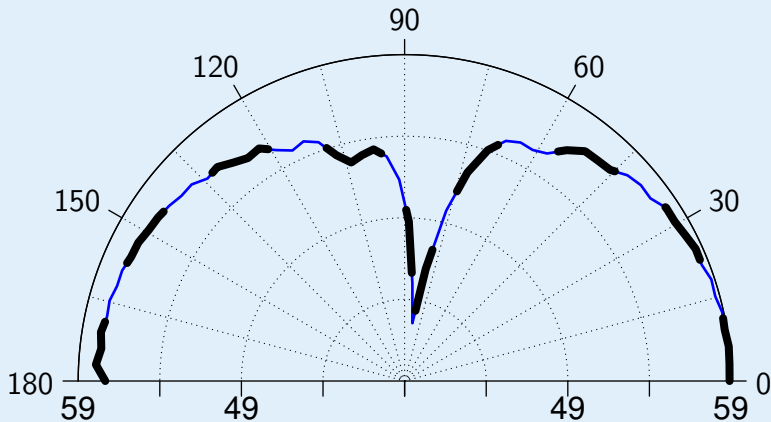
Wind turbine $k_c=0.5$



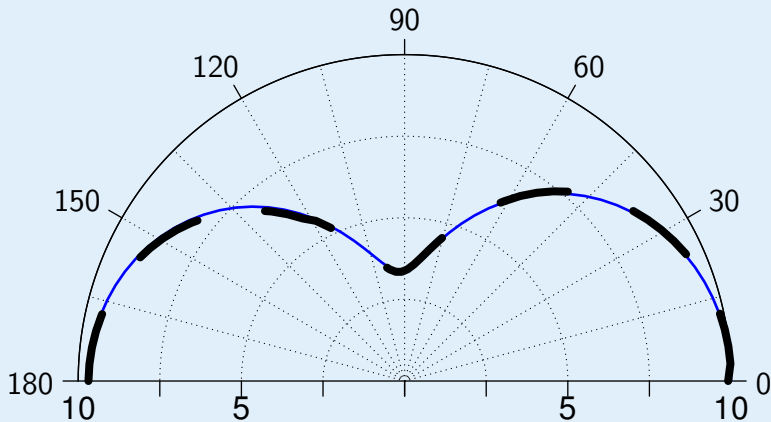
Wind turbine $k_c=5$



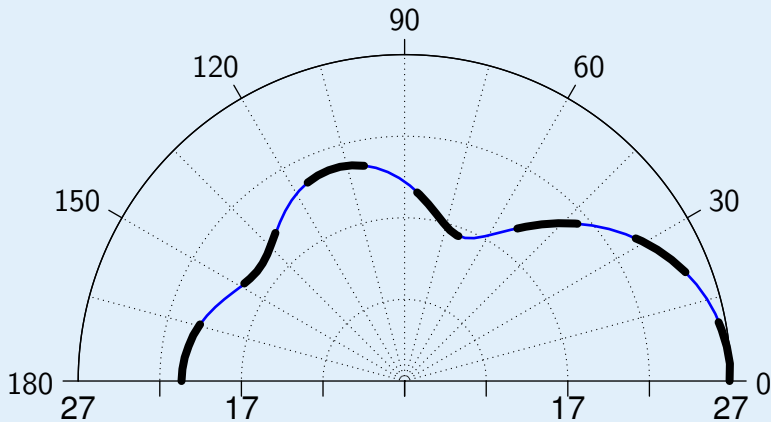
Wind turbine $kc=50$



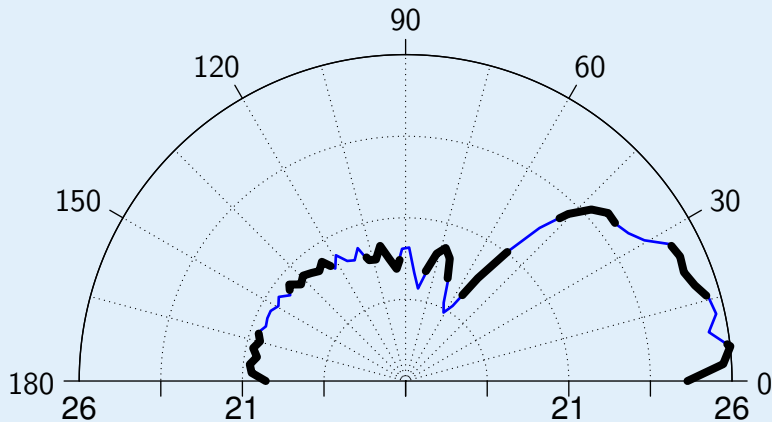
Cooling fan $kc=0.5$



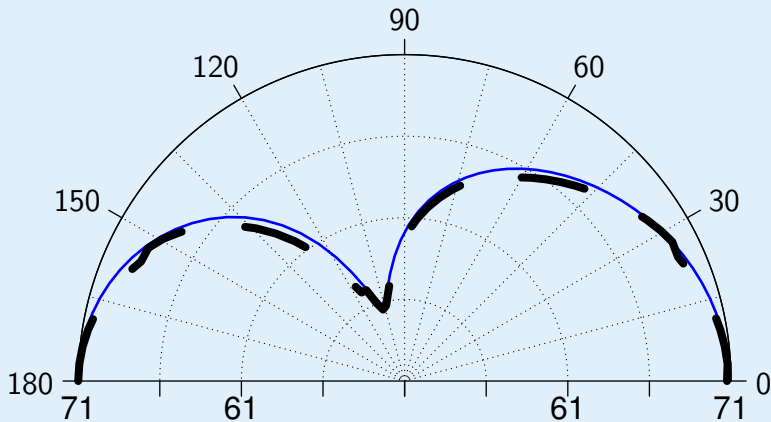
Cooling fan $kc=5$



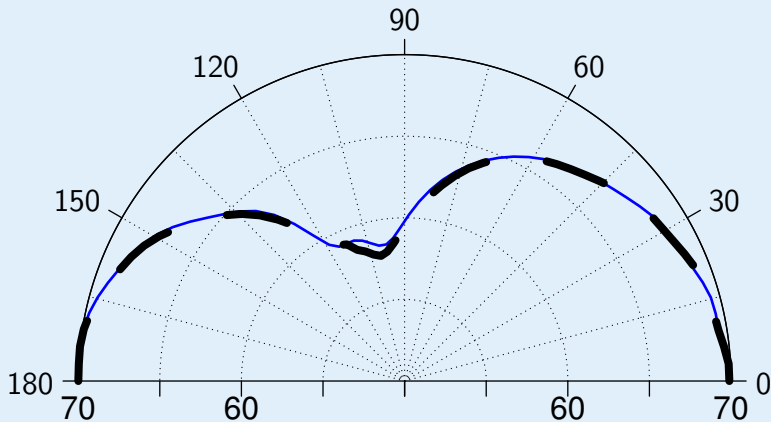
Cooling fan $kc=50$



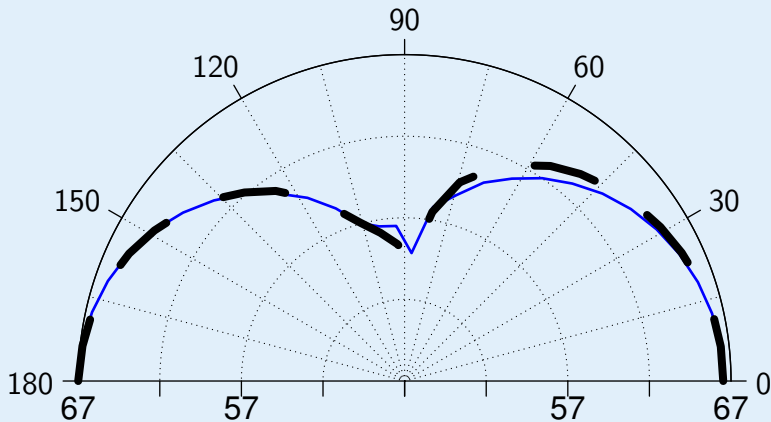
Open propellor take-off $kc=0.5$



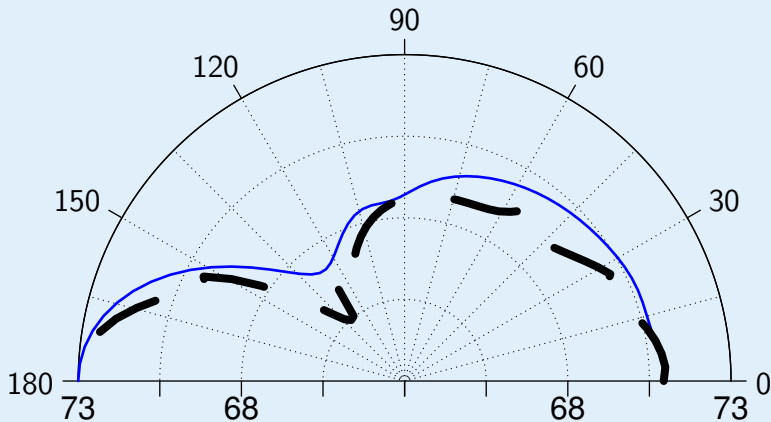
Open propellor take-off $kc=5$



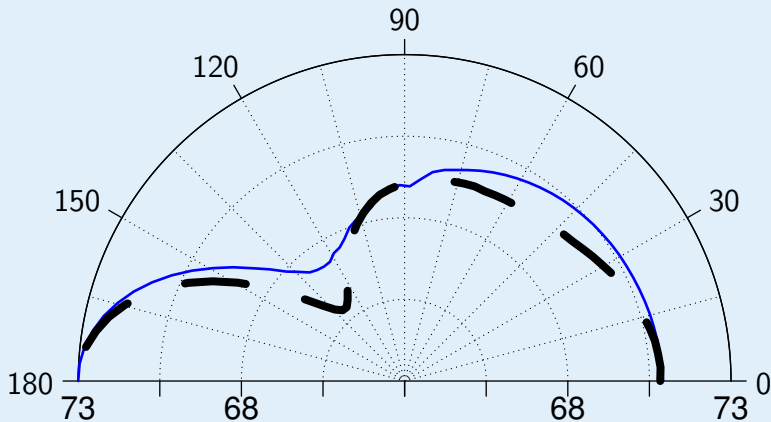
Open propellor take-off $kc=50$



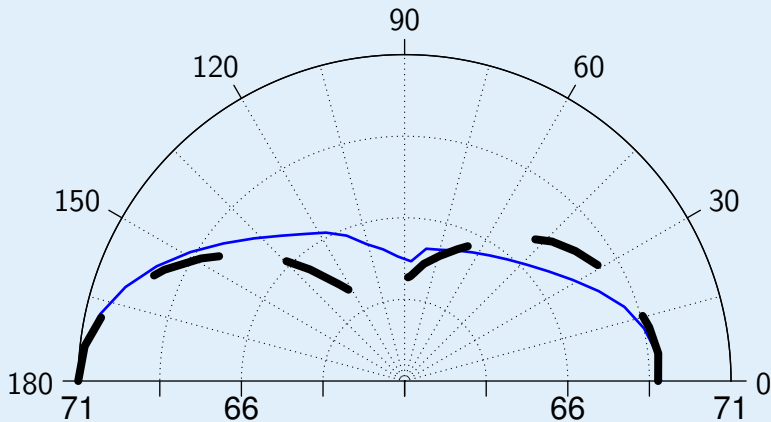
Open propeller cruise $k_c=0.5$



Open propeller cruise $k_c=5$



Open propeller cruise $k_c=50$



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2. Amiet's approach exact if $M_{ch} \leq 0.85$ and $kc > 0.5$

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1. Right exponent in Amiet's theory is 2
2. Amiet's approach exact if $M_{ch} \leq 0.85$ and $kc > 0.5$
3. Applicable to a wide range of applications